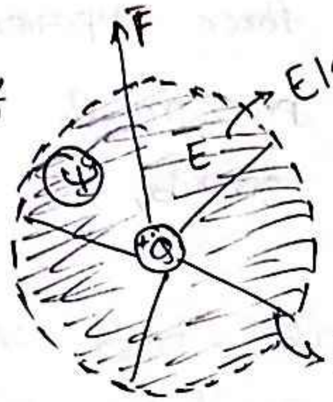
Unit-III :- Magnetostatics & Ampere's Law

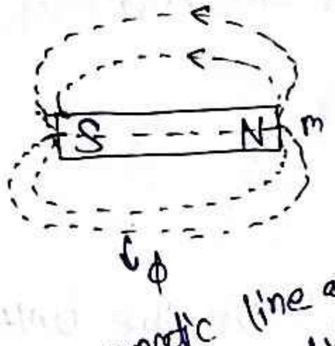
*



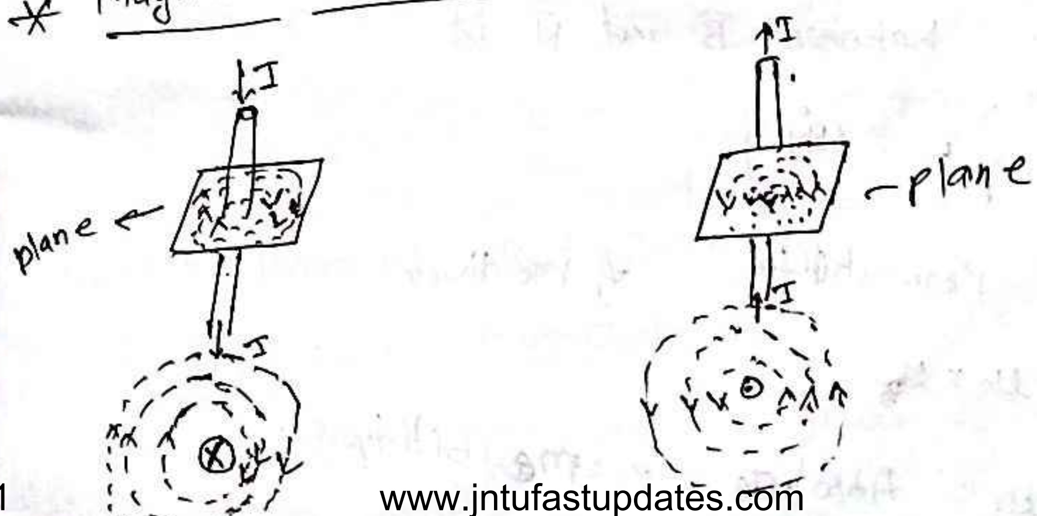
$\bar{E} = \frac{\bar{F}}{Q}$ (force per charge)

$\bar{D} = \frac{\Psi}{A}$ (flux per Area)

$\boxed{\bar{D} = \varepsilon \bar{E}}$



* Magnatic flux due to current carrying conductor.



The magnetic field intensity $\bar{H}$ at any point in a magnetic field is defined as the force experienced by a unit north pole of 1 webers of pole strength and it is denoted by H. and is given by

$$\boxed{\bar{H} = \frac{\bar{F}}{m}}$$

* $\frac{N}{wb}$ or A/m is unity of $\bar{H}$

* Magnetic flux density ($\bar{B}$):- The Total flux passing through a ~~u~~ unit area. which is ~~perpendicular~~ perpendicular of the flux is defined as magnetic flux density, and this denoted by B and is give by.

$$\boxed{\bar{B} = \phi / A} \quad wb/m^2$$

$\frac{wb}{m^2}$ is also called ~~tesla~~ tesla in the unit of flux density.

* Relation between B ~~curves~~ ~~and~~ $\bar{H}$:-

⇒ Relation between $\bar{B}$ and $\bar{H}$ is

$$\boxed{\bar{B} = \mu \bar{H}}$$

where μ = permeability of medium

$\mu = \mu_0 \times \mu_r$

where μ_r = Absolute permeability

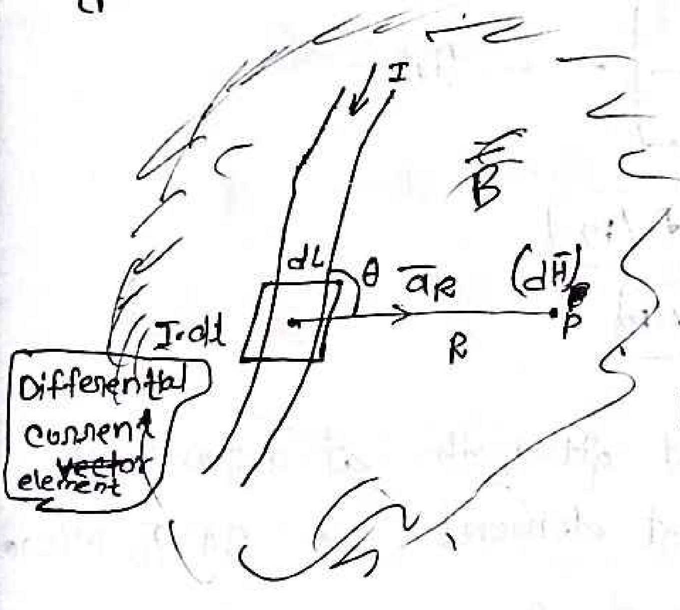
$\mu_0 = 4\pi \times 10^{-7} = H/m$

μ_r = Relative permittivity

$\mu_R = 1$ for air (or) free space.

* Biot- Savart's law:-

=> Biot- Savart's law states that the differential magnatic field strength at any point in a magnatic field due to current carying conductor is directly propotional to the product of current, Differential length (dl) and sin of angle θ and inversly propotional of square of distance between point and differential current element.



From the statement of Biot- savart's law,

$$dH \propto \frac{I\, dl \sin\theta}{R^2} \qquad \boxed{d\bar{H} = \frac{K I dl \sin\theta}{R^2}}$$

K → Constant of propotionalty.

$$K = \frac{1}{4\pi}$$

$$\boxed{dH = \frac{I dL \sin\theta}{4\pi R^2}} \quad \text{--[i]}$$

Vector form of Biot-Savart's law,

Let dL = Magnitude of $d\bar{L}$

$\bar{a}_R$ = Unit vector along $\bar{R}$ direction.

from cross product,

$$\bar{dL} \times \bar{a_R} = |\bar{dL}||\bar{a_R}| \sin\theta$$

$$\bar{dL} \times \bar{a_R} = dL \cdot 1 \cdot \sin\theta \qquad \boxed{a_R = 1}$$

$$\boxed{\bar{dL} \times \bar{a_R} = dL \sin\theta} \text{ --- (ii)}$$

Sub (2) in equ (i),

$$\boxed{d\bar{H} = \frac{I\bar{dL} \times \bar{a_R}}{4\pi R^2}} \text{ --- (iii)}$$

Total magnetic field strength $\bar{H}$ is obtained,

$$\bar{H} = \oint dH$$

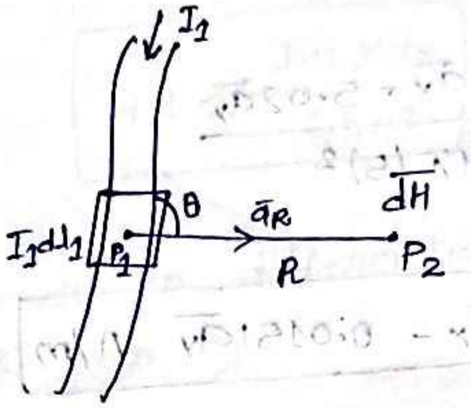
$$\boxed{\bar{H} = \oint \frac{I\bar{dL} \times \bar{a_R}}{4\pi R^2}} \text{ --- (iv)}$$

$$\boxed{\bar{H} \text{ units} = A/m}$$

$$\boxed{\bar{E} \text{ units} = V/m}$$

* Find incrimental field strength at a point P_2 due to the current element of $2\pi \bar{a_z}$ Micro Ammeter $\boxed{2\pi \bar{a_z}\ \mu Am}$ at a point P_1, the quandinates of P_1 and P_2 of $P_1(4,0,0)$ & $P_2(0,3,0)$ respectively so find incrimental field strength (or) Differential field strength.

⇒ Given data,



$I_1 dl_1 = 2\pi \bar{a}_z$

$P_1(4,0,0)$ $P_2(0,3,0)$

We have to find incremental (differential) field strength $\bar{dH}$ at P_2

$$d\bar{H}_2 = \frac{I_1 d\bar{L}_1 \times \bar{a}_R}{4\pi R^2}$$

Where $I_1 dL_1 = 2\pi \bar{a}_z$

$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} ; \quad \bar{R} = P_2 - P_1 = (0-4)\bar{a}_x + (3-0)\bar{a}_y + (0-0)\bar{a}_z$$

$$\bar{R} = -4\bar{a}_x + 3\bar{a}_y$$

$$|\bar{R}| = \sqrt{4^2 + 3^2} = 5$$

$$\bar{a}_R = \frac{-4\bar{a}_x + 3\bar{a}_y}{5} = -\frac{4\bar{a}_x}{5} + \frac{3\bar{a}_y}{5}$$

<u>Now</u>

$$I_1 d\bar{L}_1 \times \bar{a}_R = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & 2\pi \\ \frac{-4}{5} & \frac{3}{5} & 0 \end{vmatrix} = \bar{a}_x\left(0 - \frac{6\pi}{5}\right) - \bar{a}_y\left(0 + \frac{8\pi}{5}\right)$$

$$= \boxed{-\frac{6\pi}{5}\bar{a}_x - \frac{8\pi}{5}\bar{a}_y}$$

$$I_1 \, d\bar{L} \times \bar{a}_R = -3.76\bar{a}_x - 5.02\bar{a}_y$$

$$d\bar{H}_2 = \frac{-3.76\bar{a}_x - 5.02\bar{a}_y}{4\pi(5)^2}$$

$$\boxed{d\bar{H}_2 = -0.0117\bar{a}_x - 0.0151\bar{a}_y \ A/m}$$

I.V

* $\bar{H}$ due to infinitely Long Straight Conductor :-

Long Straight Conductor (or) Current Carrying Conductor,

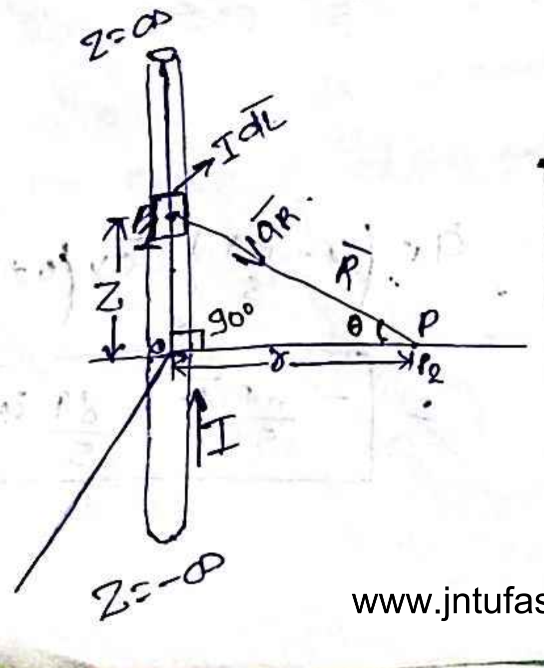
=> Let consider infinite length of conductor carries a current (I) which is placed along z-axis from $-\infty$ to ∞.

Let consider any point(P) which is distance r from origin.

Let us find magnetic field at point P.

Let assume a differential length $d\bar{L}$ at a point P_1 which is the distance z from origin.

Distance vector is $\bar{R}$ from P_1 to P and the unit vector $\bar{a}_R$



* $\overline{dH}$ at a point 'p' due to $I\overline{dL}$ is given by,

$$\boxed{d\overline{H} = \frac{I\overline{dL} \times \overline{a_R}}{4\pi R^2}} \quad --- \text{(i)}$$

* ~~dl~~ $\overline{dL}$ in differential element along $\overline{a_z}$ direction is given by

$$\boxed{\overline{dL} = dz.\overline{a_z}} \quad -- \text{[ii]}$$

* $\overline{a_R}$ is unit vector along $\overline{R}$ is given by,

$$\overline{a_R} = \frac{\overline{R}}{|R|}$$

* $\overline{R} = P_2 - P_1$ [In Cylindrical Co-ordinate System]

$$\overline{R} = (r-0)\overline{a_r} + (0-0)\overline{a_\phi} + (0-z)\overline{a_z}$$

$$\boxed{\overline{R} = r\overline{a_r} - z\overline{a_z}}$$

* $|R| = \sqrt{r^2 + z^2}$

$$\therefore \overline{a_R} = \frac{r\overline{a_r} - z\overline{a_z}}{\sqrt{r^2+z^2}}$$

$$\boxed{\overline{a_R} = \frac{r\overline{a_r}}{\sqrt{r^2+z^2}} - \frac{z\overline{a_z}}{\sqrt{r^2+z^2}}} \quad -- \text{(iii)}$$

$$\overline{dL} \times \overline{a_R} = \begin{vmatrix} \overline{a_r} & \overline{a_\phi} & \overline{a_z} \\ 0 & 0 & \text{[illegible]} \\ \frac{r}{\sqrt{r^2+z^2}} & 0 & \frac{-z}{\sqrt{r^2+z^2}} \end{vmatrix}$$

$$\bar{a_r}(0-0) - \bar{a_\phi}\left(\frac{-r\,dz}{\sqrt{r^2+z^2}}\right) + \bar{a_z}(0-0)$$

$$\boxed{d\bar{L} \times \bar{a_R} = \frac{r\,dz\,\bar{a_\phi}}{\sqrt{r^2+z^2}}} \quad --- (iv)$$

Sub equ(iv) in equ (i),

$$d\bar{H} = \frac{I\left[\dfrac{r\,dz\;\bar{a_\phi}}{\sqrt{r^2+z^2}}\right]}{4\pi R^2}$$

$$d\bar{H} = \frac{I \cdot r\,dz \cdot \bar{a_\phi}}{4\pi\left(\sqrt{r^2+z^2}\right)^2\left(\sqrt{r^2+z^2}\right)}$$

$$\boxed{d\bar{H} = \frac{I \cdot r\,dz \cdot \bar{a_\phi}}{4\pi\,(r^2+z^2)^{3/2}}} \quad -- (v)$$

From $\Delta\, P_1 O P_2$ at origin $= 90^\circ$

So, $\tan\theta = \dfrac{z}{r}$, $z = r\tan\theta$

$$\boxed{dz = r\sec^2\theta \cdot d\theta} \quad -- (vi) \;\&\; \boxed{z^2 = r^2\tan^2\theta} \; -(vii)$$

Sub (vii) & (vi) in equ(v),

$$d\bar{H} = \frac{I \cdot r \cdot r\sec^2\theta\, d\theta \cdot \bar{a_\phi}}{4\pi\,(r^2+z^2)^{3/2}}$$

$$d\bar{H} = \frac{I \cdot r \cdot r\sec^2\theta\, d\theta \cdot \bar{a_\phi}}{4\pi\,(r^2+r^2\tan^2\theta)^{3/2}}$$

$$d\overline{H} = \frac{I \cdot r \cdot \cancel{dz} \sec^2\theta \, d\theta \cdot \overline{a_\phi}}{4\pi \, r^{\cancel{3}} \sec^3\theta}$$

$$\boxed{d\overline{H} = \frac{I \cdot \cancel{dz} \cos\theta \, d\theta \cdot \overline{a_\phi}}{4\pi r^2}}$$

* Total $\overline{H}$ at point P is obtained by gntregating of above $\overline{dH}$ equ. with limits $-\infty$ to ∞

$$\overline{H} = \int_{z=-\infty}^{z=\infty} dH$$

$$\overline{H} = \int_{z=-\infty}^{z=\infty} \frac{I \cdot \cancel{dz} \cos\theta d\theta \cdot \overline{a_\phi}}{4\pi r^2}$$

$$z = r\tan\theta$$

$$\theta = \tan^{-1}\left(\frac{z}{r}\right)$$

if $z = -\infty$, $\theta = \tan^{-1}\left(\frac{-\infty}{r}\right) = \frac{-\pi}{2}$

$z = +\infty$, $\theta = \tan^{-1}\left(\frac{\infty}{r}\right) = \frac{+\pi}{2}$

$$\overline{H} = \int_{\theta=-\frac{\pi}{2}}^{\theta=\pi/2} \frac{I \cos\theta \, d\theta \cdot \overline{a_\phi}}{4\pi R}$$

$$\overline{H} = \frac{I \cdot \overline{a_\phi}}{4\pi R} \cdot \int_{-\pi/2}^{\pi/2} \cos\theta d\theta$$

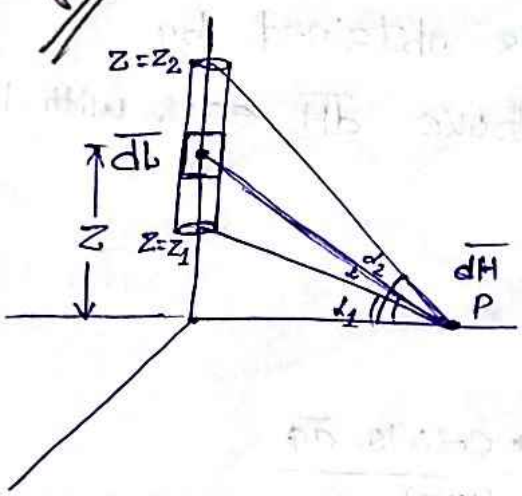
$$\overline{H} = \frac{I}{4\pi R} \overline{a_\phi} \left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

$$\overline{H} = \frac{I}{4\pi R} \overline{a_\phi} \left[\sin(\pi/2) + \sin(\pi/2)\right]$$

$$\boxed{\bar{H} = \frac{I}{2\pi r} \cdot \bar{a}_\phi}$$

* $\bar{H}$ due to Straight Conductor of Finite length.

P.U



$$\bar{dH} = \frac{I dL \times \bar{a}_R}{4\pi R^2}$$

For infinite length, of conductor

$$\bar{dH} = \frac{I \cdot \cos\theta \, d\theta \, \bar{a}_\phi}{4\pi r}$$

Total $\bar{H}$ at point P is obtained,

$$\bar{H} = \oint \bar{dH}$$

For finite length,

$$\bar{H} = \oint \frac{I \cdot \cos\theta \, d\theta \cdot \bar{a}_\phi}{4\pi r}$$

at $z = z_1$, $\theta = \alpha_1 = \tan^{-1}\left(\frac{m}{r}\right)$

at $z = z_2$, $\theta = \alpha_2 = \tan^{-1}\left(\frac{n}{r}\right)$

$$\overline{H} = \int_{\alpha_1}^{\alpha_2} \frac{I \cos\alpha \, d\alpha}{4\pi r} \cdot \overline{a_\phi}$$

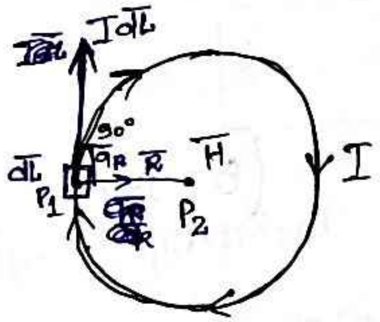
$$\overline{H} = \frac{I \cdot \overline{a_\phi}}{4\pi r} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha$$

$$\overline{H} = \frac{I \cdot \overline{a_\phi}}{4\pi r} \Big[\sin\alpha \Big]_{\alpha_1}^{\alpha_2}$$

$$\boxed{\overline{H} = \frac{I}{4\pi r} (\sin\alpha_2 - \sin\alpha_1) \overline{a_\phi}} \quad A$$

P.V

$\overline{H}$ at Centre of circular Conductor :-



* $\overline{dH}$ at point P due to $I\overline{dL}$ is given by,

$$\overline{dH} = \frac{I \overline{dL} \times \overline{a_R}}{4\pi R^2} \quad \text{---(i)}$$

* From cross product Rule,

$$\overline{dL} \times \overline{a_R} = |\overline{dL}| |\overline{a_R}| \sin\theta \cdot \overline{a_N}$$

Where $\overline{a_N}$ is unit Vector normal to plane Containing $\overline{dL}$ & $\overline{a_R}$

$$\overline{dL} \times \overline{a_R} = dL \cdot \sin\theta \cdot \overline{a_N} \quad \text{---(ii)} \quad \boxed{|\overline{a_R}| = 1}$$

Sub equ (2) in equ (i)

$$d\overline{H} = \frac{I \cdot dL \sin\theta}{4\pi R^2} \cdot \overline{a_N}$$

* H at point 'p' is obtained,

$$\overline{H} = \int d\overline{H}$$

$$\overline{H} = \int \frac{I dL \sin\theta \cdot \overline{a_N}}{4\pi R^2}$$

$$\overline{H} = \frac{I \sin\theta}{4\pi R^2} \cdot \overline{a_N} \int dL$$

$\int dL =$ circumference of circle $= 2\pi R$

$$\overline{H} = \frac{I \cdot \sin\theta \cdot \overline{a_N} \cdot 2\pi R}{4\pi R^2} \quad (\theta = 90^\circ)$$

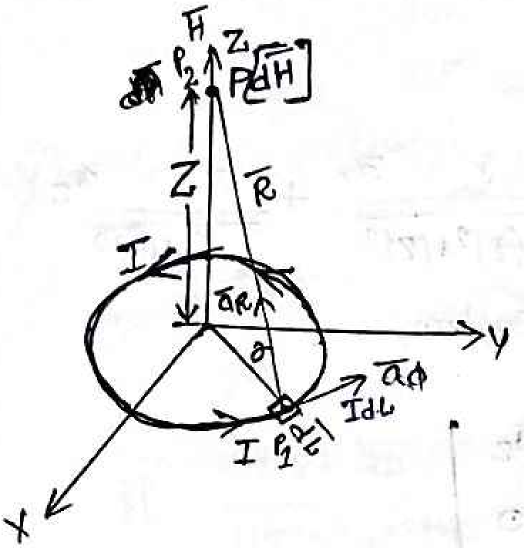
$$\boxed{\overline{H} = \frac{I \cdot \overline{a_N}}{2R}} \text{ A/m}$$

if circular ring in X-Y plane.

if circular ring in Z-plane $\boxed{\overline{a_N} = \overline{a_Z}}$

$$\boxed{\overline{H} = \frac{I \cdot \overline{a_Z}}{2R}} \text{ A/m}$$

* $\overline{H}$ on axix of circular loop.



* Circular loop is placed in xy plane [$\parallel^{el}$ to xy plane]

* According to Biot-Savart's Law $\overline{dH}$ at point 'p' due to Idl is given by

$$\boxed{\overline{dH} = \frac{I\,\overline{dL}\times \bar{a}_R}{4\pi R^2}} \quad ---(i)$$

* In Cylindrical Co-ordinate System.

$$\overline{dL} = dr.\bar{a}_r + r.d\phi.\bar{a}_\phi + dz.\bar{a}_z$$

$\overline{dL}$ is directed along $\bar{a}_\phi$ direction.

$$\overline{dL} = r d\phi.\bar{a}_\phi$$

* $\bar{a}_R$ is unit vector along $\bar{R}$ from Idl to point P is given by,

$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|}$$

$\bar{R} = P_2 - P_1 = (0-r)\bar{a}_r + (0-0)\bar{a}_\phi + (z-0)\bar{a}_z$

$$\overline{R} = -r\overline{a_r} + z\cdot\overline{a_z}$$

$$|\overline{R}| = \sqrt{(r)^2+(z)^2}$$

$$\overline{a_R} = \frac{-r\overline{a_r}+z\overline{a_z}}{\sqrt{(r)^2+(z)^2}} = \frac{-r}{\sqrt{(r)^2+(z)^2}}\overline{a_r} + \frac{z}{\sqrt{(r)^2+(z)^2}}\overline{a_z}$$

In cylindrical co-ordinate system,

$$\overline{dL}\times\overline{a_R} = \begin{vmatrix} \overline{a_r} & \overline{a_\phi} & \overline{a_z} \\ 0 & r\,d\phi & 0 \\ \frac{-r}{\sqrt{(r)^2+(z)^2}} & 0 & \frac{z}{\sqrt{(r)^2+(z)^2}} \end{vmatrix}$$

$$\overline{a_r}\left[\frac{r\,d\phi\cdot z}{\sqrt{(r)^2+(z)^2}} - 0\right] - \overline{a_\phi}\left[0\right] + \overline{a_z}\left[\frac{r^2\,d\phi}{\sqrt{(r)^2+(z)^2}}\right]$$

$$\boxed{\overline{dL}\times\overline{a_R} = \frac{r\cdot z\cdot d\phi}{\sqrt{(r)^2+(z)^2}}\cdot\overline{a_r} + \frac{r^2 d\phi}{\sqrt{(r)^2+(z)^2}}\cdot\overline{a_z}} \quad ---(ii)$$

Sub equ (ii) in equ(i),

$$d\overline{H} = \frac{I\,r\cdot z\cdot d\phi}{4\pi(r^2+z^2)^{3/2}}\cdot\overline{a_r} + \frac{I\cdot r^2 d\phi}{4\pi(r^2+z^2)^{3/2}}\cdot\overline{a_z}$$

due to radial symmetric radial components of $d\overline{H}$ get cancel each other

$\therefore$ $d\overline{H}$ has only z-components of $d\overline{H}$

Hence $$d\overline{H} = \frac{I r^2\,d\phi}{4\pi(r^2+z^2)^{3/2}}\cdot\overline{a_z}$$

$\bar{H}$ at point 'p' is obtained by

$$\bar{H} = \oint d\bar{H} = \int \frac{I r^2 \, d\phi}{4\pi(r^2+z^2)^{3/2}} \bar{a}_z$$

(*) ⇒ In cylindrical co-ordinate the range of ϕ is 0 to 2π

So

$$\bar{H} = \frac{I r^2 \bar{a}_z}{4\pi(r^2+z^2)^{3/2}} \int_0^{2\pi} d\phi$$

$$\bar{H} = \frac{I r^2 \bar{a}_z}{4\pi(r^2+z^2)^{3/2}} \left[\phi\right]_0^{2\pi}$$

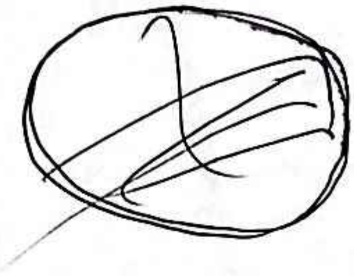
$$\boxed{\bar{H} = \frac{I r^2}{2(r^2+z^2)^{3/2}} \cdot \bar{a}_z}$$

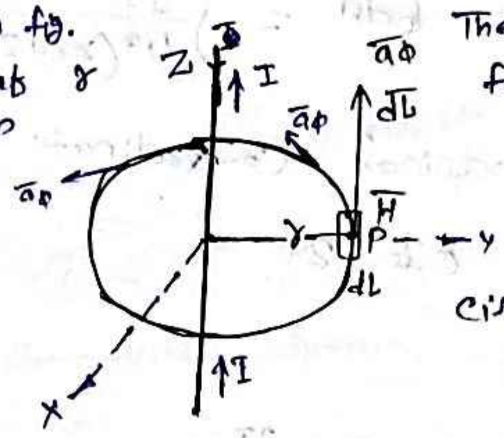
Ampere's Circutal Law :- Ampere's circuit law states that the line integral of magnatic field intensity $\bar{H}$ around a closed path is exacitely equal to the direct current enclosed by that path.

$$\boxed{\oint \bar{H} \cdot d\bar{l} = I}$$

* The Ampere's circuit law is very usefull to determin $\bar{H}$ when the current distribution is similar

* **Proof of Ampere's circuit law:-** Consider a long straight conductor carrying direct current I placed along z-axis as shown in fig. The point P is at a perpendicular distance of r from the conductor. Consider $\bar{dL}$ at point P which is in $\bar{a}_\phi$ direction, tangential to circular path at point P.





P.V

From Biot - Savart Law

* $\bar{H}$ at point P due to infinitely long straight conductor is

$$\boxed{\bar{H} = \frac{I}{2\pi r} \cdot \bar{a}_\phi}$$

* In cylindrical co-ordinate system

$\bar{dL}$ differential length acts is $\bar{a}_\phi$ direction

$$\boxed{\bar{dL} = r\,d\phi \cdot \bar{a}_\phi}$$

$$\bar{H} \cdot \bar{dL} = \frac{I}{2\pi \cancel{r}} \cdot \bar{a}_\phi \cdot \cancel{r}\,d\phi \cdot \bar{a}_\phi$$

$$\bar{H} \cdot \bar{dL} = \frac{I}{2\pi} \cdot d\phi \qquad [a_\phi \cdot a_\phi = 1]$$

integral of $\bar{H} \cdot \bar{dL}$ over entire closed path is

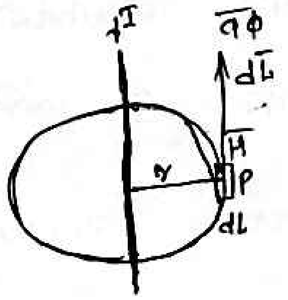
$$\oint \bar{H} \cdot \bar{dL} = \oint_{\phi=0}^{2\pi} \frac{I}{2\pi} \cdot d\phi$$

$$\oint \bar{H}\cdot d\bar{L} = \frac{I}{2\pi}\left[\phi\right]_0^{2\pi}$$

$$\oint \bar{H}\cdot d\bar{L} = \frac{I}{2\pi}\cdot 2\pi$$

$$\boxed{\oint \bar{H}\cdot dL = I}$$

(1) $\bar{H}$ due infinetly Long straight conductor



from ampere's circuit law

$$\oint \bar{H}\cdot d\bar{L} = I$$

* $\bar{H}$ at point due to infinet long straigh Conductor is
(or) $\bar{H}$ has only component in $\bar{a}_\phi$ direction is
$$\bar{H} = H_\phi \cdot \bar{a}_\phi$$

* In Cylindrical Co-ordinate System, $d\bar{L}$ differential length acts in $\bar{a}_\phi$ direction.

$$d\bar{L} = r\,d\phi \cdot \bar{a}_\phi$$

$$\cancel{\bar{H}\cdot d\bar{L} = H_\phi \cdot H_\phi}$$

$$\bar{H}\cdot d\bar{L} = H_\phi \cdot \bar{a}_\phi \cdot r\,d\phi \cdot \bar{a}_\phi$$

$$\bar{H}\cdot d\bar{L} = H_\phi \cdot r\,d\phi \quad [\bar{a}_\phi\cdot\bar{a}_\phi = 1]$$

Sub $H \cdot \bar{dL}$ in equ(1),

$$\int_{\phi=0}^{2\pi} H\phi\, r \cdot d\phi = I$$

$$H\phi\, r \cdot [\phi]_0^{2\pi} = I \qquad \boxed{H\phi\, r\, 2\pi = I}$$

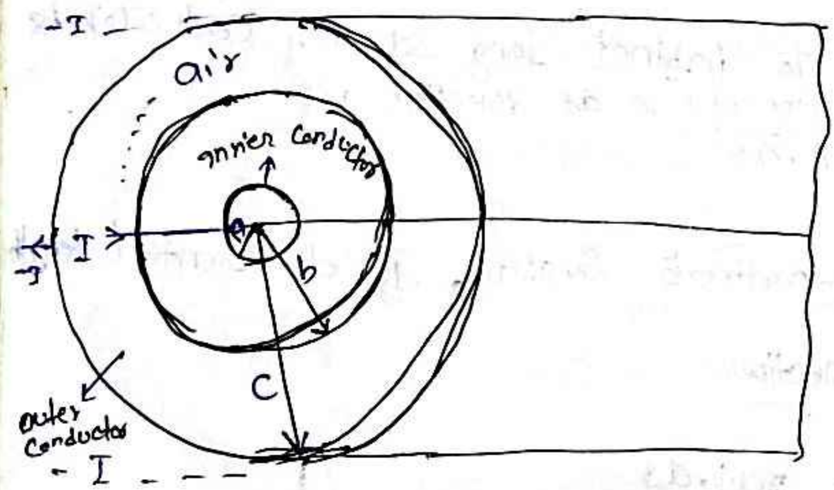
$$H\phi = \frac{I}{2\pi r}$$

Hence $\bar{H}$ at any point in

$$\bar{H} = H\phi \cdot \bar{a}\phi$$

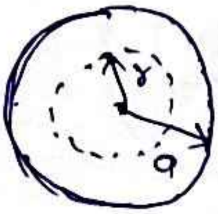
$$\boxed{\bar{H} = \frac{I}{2\pi r} \cdot \bar{a}\phi}$$

[2] $\bar{H}$ due to co-axial cable :-



Case (i)

$r<a$ Inner part of inner Conductor



According to Ampere's Circuital law

$$\oint H \cdot d\bar{L} = I$$

* I' is current passing through part of inner conductor current I having area πr^2 & Area of inner Conductor is πa^2

* Current density $J = \frac{I}{A}$

$$\int_{\phi=0}^{2\pi} H_{1\phi} \cdot r\, d\phi = I'$$

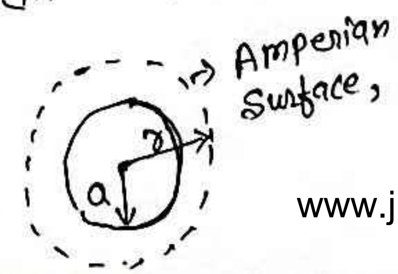
$$H_{1\phi} \cdot r \left[\phi\right]_0^{2\pi} = I'$$

$$H_{1\phi}\, 2\pi r = I'$$

$$H_{1\phi}\, 2\pi r = \frac{I r^2}{a^2}$$

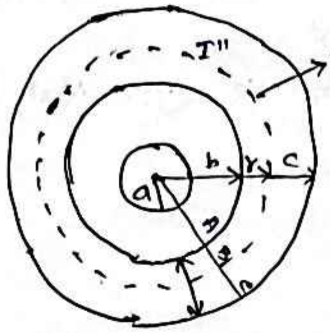
$$\boxed{H_{1\phi} / H_1 = \frac{I r \cdot \bar{a}_\phi}{2\pi a^2}} \qquad (\bar{H} \propto r)$$

Case (ii):- Outer part of inner conductor :-



$$\boxed{\bar{H} = \frac{I}{2\pi r}\, \bar{a}_\phi}$$

Case (iii):- $b < r < c$



According to Ampere's Circuit law,

$$\oint \bar{H} \cdot \bar{dL} = I'''$$

$$\frac{I''}{\pi(r^2-b^2)} = \frac{-I}{\pi(c^2-b^2)}$$

$$I'' = \frac{-I(r^2-b^2)}{(c^2-b^2)}$$

Total Current $I''' = I'' + I$

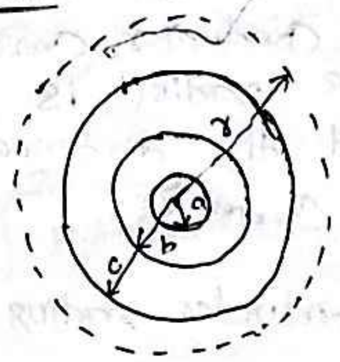
$$\boxed{I''' = I - \frac{I(r^2-b^2)}{c^2-b^2}}$$

$$I''' = I\left[\frac{c^2-\cancel{b^2}-r^2+\cancel{b^2}}{c^2-b^2}\right]$$

$$I''' = I\left[\frac{c^2-r^2}{c^2-b^2}\right]$$

$$\boxed{\bar{H}_{3\phi} = \frac{I(c^2-r^2)}{2\pi r(c^2-b^2)}}$$

Case (iv)



* Ampere's circuit law,

$$\oint \overline{H} \cdot dL = I^{IV}$$

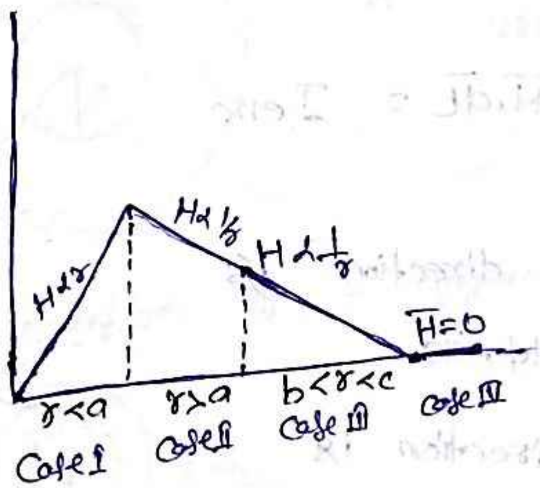
$$I^{IV} = I - I = 0$$

$$\boxed{\int \overline{H} \cdot dL = 0}$$

The magnatic field intensity in outside the Conductor (or) outer conducter of the Coaxial cable is zero.

$$\boxed{\overline{H} = 0}$$

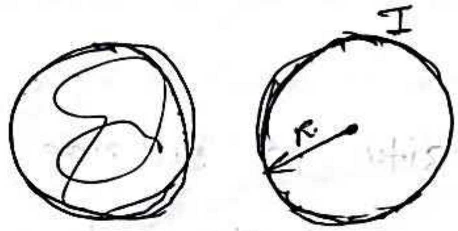
* Varriation of $\overline{H}$ against 'r' :-



Obtain the expression for $\overline{H}$ in all the region if a cylindrical conductor carries a direct current I, and its radius is R m the variation of $\overline{H}$ against the distance R from the centre of the conductor.

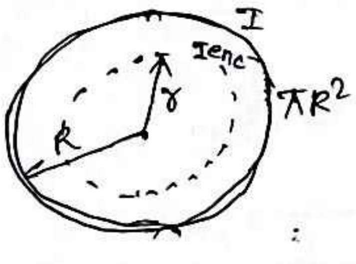
=> Let assumed cylindrical conducter radius (R)

(i) Cylindrical conductor,



2 cases is there because in Co-axial cable one sphere is there,

<u>Case (i):-</u> $r < R$ (amperian closed path in side of conductor.)



* Ampine's circuit law

$$\oint \overline{H} \cdot \overline{dL} = I_{enc}$$

* $\overline{H}$ is along $\overline{a_\phi}$ direction is

$$\overline{H} = H_\phi \cdot \overline{a_\phi}$$

* $\overline{dL}$ along $\overline{a_\phi}$ direction is

$$\overline{dL} = r \cdot d\phi \cdot \overline{a_\phi}$$

* Ienc is current enclosed by closed path is

$$\frac{I_{enc}}{\pi r^2} = \frac{I}{\pi R^2}$$

$$\boxed{I_{enc} = \frac{I r^2}{R^2}}$$

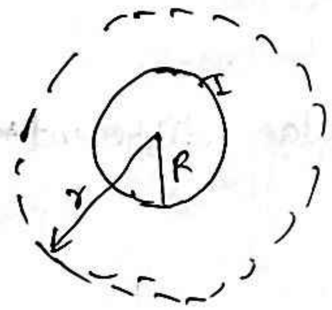
$$\int_0^{2\pi} H\phi \cdot r \cdot d\phi = \frac{I r^2}{R^2}$$

$$H\phi \cdot r \cdot 2\pi = \frac{I r^2}{R^2}$$

$$H\phi = \frac{I \cdot r}{2\pi R^2}$$

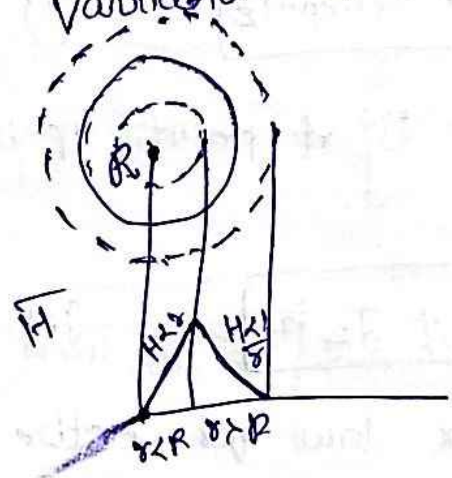
$$\boxed{\overline{H} = \frac{I \cdot r}{2\pi R^2} \cdot \overline{a\phi}} \quad (\overline{H} \propto r)$$

Case - II :- $r > R$ (Amprian closed path outside the conductor.)



$$\overline{H} = \frac{I}{2\pi R} \cdot \overline{a_r} \qquad \overline{H} \propto \frac{1}{r}$$

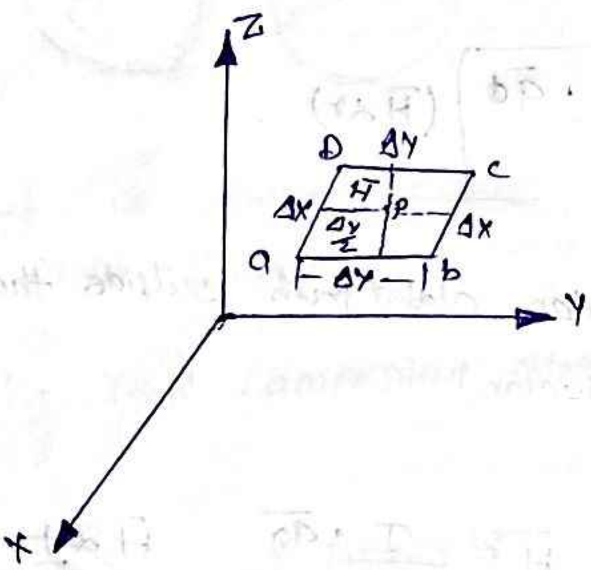
Varriation of $\overline{H}$ aggainst ϕ 'r'



* Curl Concept:-

* In electrostatics, Gauss law is applied to the
(1) Differential Volume element to develop the concept of divergence.
(2) In magnetostatics. Amperes circuital law is to be applied to the differential surface element to developed the concept of curl.

Curl :-



* $\overline{H}$ at centre of rectangular differential surface element is

$$\boxed{\overline{H} = H_{x_0} \cdot \overline{a}_x + H_{y_0} \overline{a}_y + H_{z_0} \cdot \overline{a}_z} \quad \text{--- (i)}$$

* Total current density $\overline{J}$ at point 'P' is given by

$$\boxed{\overline{J} = J_x \overline{a}_x + J_y \overline{a}_y + J_z \overline{a}_z} \quad \text{--- (ii)}$$

* Applying Ampere's circuit law for entire differential surface element abcd

$$\oint(\bar{H}\cdot\bar{dL})_{abcd} = (\bar{H}\cdot\bar{dL})_{a-b} + (\bar{H}\cdot\bar{dL})_{b-c} + (\bar{H}\cdot\bar{dL})_{c-d} + (\bar{H}\cdot\bar{dL})_{d-a}$$

(i) Along a-b

* $\bar{H}$ acts in +ve y- direction

$$\bar{H} = \bar{H}_y \cdot \bar{a}_y$$

* $\bar{dL}$ act in y- direction,

$$\bar{dL} = \Delta y \cdot \bar{a}_y$$

$$\bar{H}\cdot\bar{dL} = \bar{H}_y \cdot \Delta y \, (\bar{a}_y \cdot \bar{a}_y)$$

$$(\bar{a}_y \cdot \bar{a}_y = 1)$$

$$\boxed{\bar{H}\cdot\bar{dL} = H_y \cdot \Delta y}$$

* Where, H_y is intensity along a-b can be expressed as

$$H_y = H_{y_0} + \left[\left[\begin{matrix}\text{Differential } \bar{H}_y \text{ w.r.to } x \\ \text{in +ve } x\text{-axis}\end{matrix}\right] \times \left[\begin{matrix}\text{Distance at length} \\ \text{a-b from point}\end{matrix}\right]\right]$$

$$H_y = H_{y_0} + \left(\frac{\partial H_y}{\partial x} \cdot \frac{\Delta x}{2}\right)$$

$$(\bar{H}\cdot\bar{dL})_{a\cdot b} = \left[H_{y_0} + \left(\frac{\partial H_y}{\partial x} \cdot \frac{\Delta x}{2}\right)\right] \Delta y$$

$$\boxed{(\bar{H}\cdot\bar{dL})_{a\cdot b} = \left[H_{y_0}\,\Delta y + \frac{\partial H_y}{\partial x} \cdot \frac{\Delta x\,\Delta y}{2}\right]} \quad \text{--- (3)}$$

(2) along b-C

* $\overline{H}$ acts in -Ve X-direction,

$$\overline{H} = -H_x \cdot \overline{a_x}$$

* $\overline{dL}$ act in X-direction

$$\overline{dL} = \Delta x \cdot \overline{a_x}$$

$$(\overline{H} \cdot \overline{dL})_{b-c} = -H_x \cdot \Delta x \qquad (a_x \cdot a_x) = 1$$

* Where H_x is intensity along b-c can be expressed as,

$$H_x = H_{x_0} + \left[\begin{pmatrix} \text{Differential } H_x \text{ w.r.to} \\ y \text{ in +ve y-axis} \end{pmatrix} \times \begin{pmatrix} \text{Distance of length} \\ \text{b-c from point P} \end{pmatrix} \right]$$

$$H_x = H_{x_0} + \left(\frac{\partial H_x}{\partial y} \cdot \frac{\Delta y}{2} \right)$$

$$(\overline{H} \cdot \overline{dL})_{b-c} = -\left[H_{x_0} + \left(\frac{\partial H_x}{\partial y} \cdot \frac{\Delta y}{2} \right) \right] \Delta x$$

$$\boxed{(\overline{H} \cdot \overline{dL})_{b-c} = -H_{x_0} \Delta x - \frac{\partial H_x}{\partial y} \cdot \frac{\Delta x \Delta y}{2}} \quad \text{---} (4)$$

(3) along C-D

$\overline{H}$ acts in -Ve Y-direction

$$\overline{H} = -H_y \cdot \overline{a_y}$$

* $\bar{dL}$ acts in y- direction is

$$\bar{dL} = \Delta y . \bar{a}_y$$

$$* (\bar{H} . \bar{dL})_{c-d} = -H_y \Delta y \quad (\bar{a}_y . \bar{a}_y = 1)$$

Where H_y is intensity along cd in -ve y-direction can be expressed as

$$H_y = H_{y0} - \left[\begin{pmatrix} \text{Differentiating } H_y \\ \text{w.r.t } x \text{ in y dir} \end{pmatrix} \begin{pmatrix} \text{distance of length cd} \\ \text{from point p.} \end{pmatrix} \right]$$

$$H_y = H_{y0} - \frac{\partial H_y}{\partial x} . \frac{\Delta x}{2}$$

$$(\bar{H} . \bar{dL})_{c-d} = - \left[H_{y0} - \frac{\partial H_y}{\partial x} . \frac{\Delta x}{2} \right] \Delta y$$

$$\boxed{(\bar{H} . \bar{dL})_{c-d} = -H_{y0} \Delta y + \frac{\partial H_y}{\partial x} \frac{\Delta y . \Delta x}{2}} \leftarrow - - (v)$$

(4) along D -A

$\bar{H}$ exists on +ve x direction.

$$\bar{H} = \bar{H}_x . \bar{a}_x$$

* $\bar{dL}$ acts in x- direction.

$$\bar{dL} = \Delta x . \bar{a}_x$$

$$(\bar{H} . \bar{dL})_{D-A} = \bar{H}_x . \Delta x (\bar{a}_x . \bar{a}_x)$$

$$(\bar{H} . \bar{dL})_{D-A} = \bar{H}_x . \Delta x (1)$$

Where H_x is intensity along d-a in +ve x-direction can be expressed as.

$$H_x = H_{x0} \pm \left[\begin{pmatrix} \text{differentiating } H_x \\ \text{w.r.t } y \text{ in} \\ x\text{-direction} \end{pmatrix} \begin{pmatrix} \text{Distance of length} \\ \text{d-a from point } p \end{pmatrix} \right]$$

$$H_x = H_{x0} - \frac{\partial H_x}{\partial y} \cdot \frac{\Delta x}{2}$$

$$(\bar{H}\cdot d\bar{L})_{d-a} = \left[H_{x0} - \frac{\partial H_x}{\partial y} \frac{\Delta y}{2} \right] \Delta x$$

$$\boxed{(\bar{H}\cdot d\bar{L})_{d-a} = H_{x0}\,\Delta x - \frac{\partial H_x}{\partial y} \frac{\Delta y \Delta x}{2}} \quad --- \text{(vi)}$$

Now from equ (3) (4) (5) & (6),

$$\oint(\bar{H}\cdot d\bar{L})_{abcd} = \cancel{H_{y0}\Delta y} + \frac{\partial H_y}{\partial x} \cdot \frac{\Delta x \Delta y}{2} - \cancel{H_{x0}\Delta x} - \frac{\partial H_x}{\partial y} \frac{\Delta x \Delta y}{2} - \cancel{H_{y0}\Delta y} + \frac{\partial H_y}{\partial x} \frac{\Delta x \Delta y}{2} + \cancel{H_{x0}\Delta x} - \frac{\partial H_x}{\partial y} \frac{\Delta y \Delta x}{2}$$

$$\oint(\bar{H}\cdot d\bar{L})_{abcd} = \frac{\partial H_y}{\partial x} \cdot \Delta x \Delta y - \frac{\partial H_x}{\partial y} \Delta x \Delta y$$

$$\oint(\bar{H}\cdot d\bar{L})_{abcd} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \Delta x \Delta y$$

According to Ampere's circuital law.

$$\oint \bar{H} \cdot d\bar{L} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \Delta x \Delta y = I_{enc}$$

<u>Where,</u>

Ienc is current enclosed by differential Surface element abcd

Ienc = Current density × Area of differential surface element.

* $I_{enc} = J_z \cdot \Delta x \Delta y$

$$\bar{H} \cdot d\bar{L} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \Delta x \Delta y = J_z \, \Delta x \Delta y$$

$$\underset{\Delta x \Delta y \to 0}{Lt} \oint \frac{\bar{H} \cdot d\bar{L}}{\Delta x \Delta y} = J_z = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)$$

$\boxed{J_z = \text{Current density}}$

Where $J_z \rightarrow$ Current density normal differential Surface

$$\boxed{J_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}} \quad \text{--- (VII)}$$

Similarly

* If differential surface element is placed in parallel to YZ plane then

$$\boxed{J_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}} \quad \text{--- (VIII)}$$

If differential surface element placed in parallel to zx plane.

* Curl in
[i] Cartesian

then $$\boxed{J_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}} \quad \text{--- (ix)}$$

$\nabla \times$

Sub equ (7) (8) & (9) in equ (2)

$$\boxed{\bar{J} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\bar{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\bar{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\bar{a}_z = \text{Curl } \bar{H}}$$

$$\boxed{\therefore J = \nabla \times \bar{H} = \text{Curl } \bar{H}}$$

Point form of Ampere's Circuital law, (or)

$\nabla \times \bar{H}$

$\text{div } \bar{D} = \nabla \cdot \bar{D} = \rho_v$

one of the maxwell equ in magnetostatics.

(iii) S

∇

*

* Curl in Various Co-ordinate System:-

[i] Cartesian System.

$$\nabla \times \bar{H} = \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\bar{a}_x + \left[\frac{\partial H_x}{\partial H_z} - \frac{\partial H_z}{\partial x}\right]\bar{a}_y + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]\bar{a}_z$$

(ii) Cylindrical System,

$$\nabla \times \bar{H} = \left[\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right]\bar{a}_r + \left[\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}\right]\bar{a}_\phi + \left[\frac{1}{r}\frac{\partial (r H_\phi)}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial \phi}\right]\bar{a}_z$$

(iii) Spherical Co-ordinate System.-

$$\nabla \times \bar{H} = \frac{1}{r\sin\theta}\left[\frac{\partial H_\phi \sin\theta}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial (r H_\phi)}{\partial r}\right]a_\theta + \frac{1}{r}\left[\frac{\partial (r H_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\bar{a}_\phi$$

* A $\bar{H}$ due to a current source is given by

$$\bar{H} = [y\cos(ax)]\bar{a}_x + [y + e^x]\bar{a}_z .$$

Find the current density over the y-z plane.

(I) find $\bar{J}$

$$\bar{J} = \nabla\times\bar{H} \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\cos(ax) & 0 & y+e^x \end{vmatrix}$$

$$\bar{J} = \nabla\times\bar{H} = \left[\frac{\partial(y+e^x)}{\partial y} - 0\right]\bar{a}_x - \left[\frac{\partial(y+e^x)}{\partial x} - \frac{\partial(y\cos ax)}{\partial z}\right]$$

$$\bar{a}_y + \left[0 - \frac{\partial}{\partial y}(y+\cos ax)\right]\bar{a}_z$$

$$\nabla\times\bar{H} = [1+0-0]\,\bar{a}_y - [0+e^x]-0]\,\bar{a}_y + [-\cos ax]\,\bar{a}_z$$

$$\nabla\times\bar{H} = 1\,\bar{a}_x - e^x\,\bar{a}_y - \cos ax\cdot\bar{a}_z$$

in yz plane $(x=0)$

$$\boxed{\bar{J} = \nabla\times\bar{H} = \bar{a}_x - \bar{a}_y - \bar{a}_z \quad A/m^2}$$

* annother method:

$$\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\bar{a}_x + \left[\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial x}\right]\bar{a}_y + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]\bar{a}_z.$$

$$\bar{H} = H_x\,\bar{a}_x + H_y\cdot\bar{a}_y + H_z\cdot\bar{a}_y$$

$$H_x = y\cos ax, \quad H_y = 0, \quad H_z = y+e^x$$

$$\nabla\times\bar{H} = J = [(1+0)-0]\,\bar{a}_x + [0-1]\,\bar{a}_y$$

$$+ [0 - \cos ax\cdot 1]\,\bar{a}_z$$

$\nabla \times \bar{H} = \bar{J} = \bar{a}_x - \bar{a}_y - \cos ax \; \bar{a}_z$

$J = \bar{a}_x - \bar{a}_y - \bar{a}_z$

Given general vector $\bar{A} = \sin 2\phi \cdot \bar{a}_\phi$ in a cylindrical co-ordinate system find curl of $\bar{a}$ at $[2, \pi/4, 0]$.

~~given~~ $\bar{A} = \sin 2\phi \cdot \bar{a}_\phi = A_r = 0, \; A_\phi = \sin 2\phi, \; A_z = 0$

* cylindrical co-ordinate system.

$\nabla \times \bar{A} =$ find Curl of $\bar{A}$

$\bar{H} = \bar{A}$ (in cylindrical co-ordinate system).

Curl of $\bar{A} = \nabla \times \bar{A} =$

$$\left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \cdot \bar{a}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \bar{a}_\phi + \left(\frac{1}{r} \cdot \frac{\partial (r \cdot A_\phi)}{\partial r} - \frac{1}{r} \cdot \frac{\partial A_r}{\partial \phi} \right) \cdot \bar{a}_z$$

$$\nabla \times \bar{A} = \left(\frac{1}{r} \cdot 0 - 0 \right) \cdot \bar{a}_r + (0 - 0) \bar{a}_\phi + \left(\frac{1}{r} \cdot \frac{\partial}{\partial r} (r \cdot \sin 2\phi) - \frac{1}{r} \cdot 0 \right) \bar{a}_z$$

$$\nabla \times \bar{A} = \frac{1}{r} \cdot \sin 2\theta \cdot \bar{a}_z \text{ at point } (2, \pi/4, 0)$$

$$\nabla \times \bar{A} = \frac{1}{2} \sin 2\left(\frac{\pi}{4}\right) \cdot \bar{a}_z$$

$$\boxed{\nabla \times \bar{A} = 0.5\, \bar{a}_z}$$

* Given that the general vector $\bar{H} = 2.5\bar{a}_\theta + 5\bar{a}_\phi$ in Spherical Co-ordinate find curl of $\bar{H}$ at $\left(2, \frac{\pi}{6}, 0\right)$ =

$$\boxed{\text{Given that } H_r = 0, H_\theta = 2.5, H_\phi = 5}$$

$\Rightarrow$ $\nabla \times \bar{H} =$ (or) curl of $\bar{H}$ =

$$\nabla \times \bar{H} = \frac{1}{r\sin\theta}\left[\frac{\partial H_\phi \sin\theta}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right] \cdot \bar{a}_r +$$

$$\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r}\right] \cdot \bar{a}_\theta +$$

$$\frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right] \cdot \bar{a}_\phi$$

$$\cancel{\nabla \times \bar{H} = \frac{1}{r\sin\theta}\left[0 - 0\right] + \frac{1}{r}\left[0 - \right.}$$

$$\nabla \times H = \frac{1}{r\sin\theta}\left(\frac{\partial(5\sin\theta)}{\partial\theta} - 0\right)\bar{a}_r + \frac{1}{r}\left(0 - \frac{\partial}{\partial r}(r \cdot 5)\right)\bar{a}_\theta +$$

$$\frac{1}{r}\left(\frac{\partial(r \cdot 2.5)}{\partial r} - 0\right)\bar{a}_\phi$$

$$= \frac{1}{r\sin\theta} \cdot 5\cos\theta \cdot \bar{a}_r - \frac{1}{r} 5 \cdot 1\, \bar{a}_\theta + \frac{1}{r} 1 \cdot 2.5\, \bar{a}_\theta$$

$$\nabla \times \bar{H} = \frac{5}{r} \cot\theta \cdot \bar{a}_r - \frac{5}{r} \bar{a}_\theta + \frac{2.5}{r} \bar{a}_\theta$$

$\nabla \times \bar{H}$ at point $(2, \pi/6, 0) = (r, \theta, \phi)$

$$\boxed{\nabla \times \bar{H} = \frac{5}{2} \cot(\pi/6) \cdot \bar{a}_r - \frac{5}{2} \cdot \bar{a}_\theta + \frac{2.5}{2} \cdot \bar{a}_\phi}$$

$$\boxed{\nabla \times \bar{H} = 4.33\, \bar{a}_r - 2.5\, \bar{a}_\theta + 1.25\, \bar{a}_\phi}$$

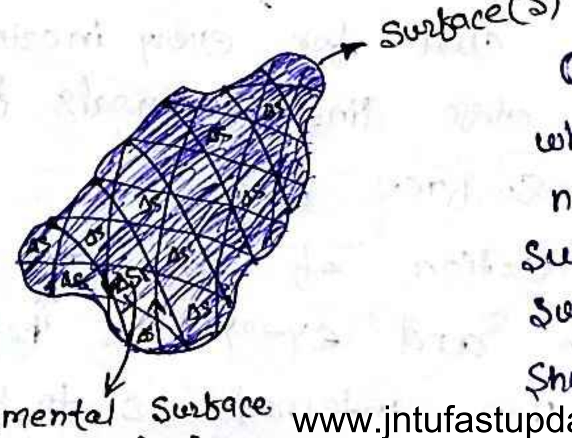
Stoke's theorem :-

Stoke's theorem relates the line integral to surface integral

Statement :- The line integral of a vector $\bar{A}$ around a closed path is equal to the integral of curl of $\bar{A}$ over the open surface inclosed by the closed path.

Stoke's theorem is applicable for open surface enclosed by the given closed path.

Proof of Stoke's theorem:-



Consider a surface S which is splitted into number of incremental surface, Each incremental surface is having ΔS as shown in fig.

* Applying curl for any incremental surface is given

$$\boxed{(\nabla \times H)_N = \frac{\oint \overline{H} \cdot d\overline{L}_{\Delta S}}{\Delta S}} \quad ---(i)$$

Where N is normal to incremental surface and

$d\overline{L}_{\Delta S}$ is perimeter of incremental surface 'ΔS'

*

$$\boxed{\oint \overline{H} \cdot d\overline{L}_{\Delta S} = (\nabla \times \overline{H})_N \cdot \Delta S} \quad ---(ii)$$

~~Where~~ ~~$(\nabla\times\overline{H})_N$~~

(The curl of $\overline{H}$ in normal direction

$(\nabla\times\overline{H})_N$ is) the dot product of

$(\nabla\times\overline{H})\cdot\overline{a}_N$ curl of $\overline{H}$ with a unit vector $\overline{a}_n$ normal to the surface ΔS that is,

$$* \quad (\nabla\times\overline{H})_N = (\nabla\times\overline{H})\cdot\overline{a}_N \quad --- (iii)$$

Substitute equ (iii) in equ (ii)

therefore,

$$\oint \overline{H}\cdot d\overline{L}_{\Delta S} = (\nabla\times\overline{H})\cdot\overline{a}_N\cdot\Delta S$$

therefore,

$$\boxed{\oint \overline{H}\cdot d\overline{L}_{\Delta S} = (\nabla\times\overline{H})\cdot\overline{\Delta S}} \quad ---(iv) \quad (\overline{\Delta S} = \Delta S\cdot\overline{a}_n)$$

(To obtained total curl for every incremental surface and the close line integrals for each incremental surface ΔS)

Hence summation of all closed line integral for each and every ΔS hence i a single close line integral is to be obtained for total surface (S). Hence equ (iv) becomes,

$$\oint_L \bar{H}\cdot d\bar{L} = \int_S (\nabla \times \bar{H})\cdot d\bar{S}$$

$$* \quad \boxed{\oint_L \bar{H}\cdot d\bar{L} = \int_S (\nabla \times \bar{H})\cdot dS} \quad --- \ (5) \ \text{(Stoke's theorem)}$$

Where $d\bar{L}$ is perameter of total surface 'S'.

~~The~~ Thus the line integral can be expressed surface integral which proves stoke's theorem.

Note:- Stoke's theorem is aplicable for open surface but not for closed surface. When it is aplicable for open surface its output became zero.

Magnatic flux density & Magnatic flux :-

Magnatic flux

$$\boxed{\bar{B} = \frac{\phi}{A}} \quad Wb/m2 \ (or) \ Tesla$$

* If flux passes through surface area & making some angle with plane then flux crossing

through Surface area is given by.

$$\boxed{\phi = \int_S \bar{B} \cdot \bar{ds}}$$

Where

$\bar{ds}$ = open Surface area

* For closed Surface area having Volume.

∴ Total flux in a closed Surface is zero.

$$\boxed{\phi = \int_S \bar{B} \cdot \bar{ds} = 0}$$

* Applying divergence for closed

* A radial field $\overline{H} = 2.39 \times 10^6/r \cos\phi . \overline{a_r}$ A/m exists in free space find the magnetic flux crossing the surface defined by $0 \leq \phi \leq \pi/4$ and $0 \leq z \leq 1m$

$\Rightarrow$ Given data,

$$\overline{H} = \frac{2.396 \times 10^6}{r} \cos\phi . \overline{a_r} \text{ A/m}$$

$0 \leq \phi \leq \pi/4$

$0 \leq z \leq 1m$

By observing limits we get that is cylindrical co-ordinate

find magnetic flux crossing cylindrical surface

$$\phi = \int_S \overline{B} . \overline{ds}$$

$\overline{B} = \mu \overline{H}$

$\overline{H}$ is given, $= \frac{2.396 \times 10^6}{r} \cos\phi . \overline{a_r}$

Now

$$B = \mu \times \frac{2.396\times10^6}{r} \times \cos\phi \cdot \bar{a}_r \quad (\mu = \mu_0)$$

$$B = \mu_0 \times \frac{2.396\times10^6}{r} \times \cos\phi \cdot \bar{a}_r$$

Where $\boxed{\mu_0 = 4\pi\times10^{-7}}$

$$B = 4\pi\times10^{-7} \times \frac{2.396\times10^6}{r} \times \cos\phi \cdot \bar{a}_r$$

$$\boxed{B = \frac{3\cos\phi \cdot \bar{a}_r}{r}}$$

* ds is normal to $\bar{a}_r$ in Cylindrical co-ordinate System.

$$\boxed{d\bar{S} = r\,d\phi\,dz \cdot \bar{a}_r}$$

Now

$$\phi = \int_S \frac{3\cos\phi \cdot \bar{a}_r}{r} \cdot r\,d\phi\,dz \cdot \bar{a}_r$$

There is Surface Integral (S) So Double Int is there.

$$\boxed{\bar{a}_r \cdot \bar{a}_r = 1}$$

$$\phi = \int_{z=0}^{1}\int_{\phi=0}^{\pi/4} 3\cos\phi \cdot d\phi \cdot dz$$

$$\phi = 3\int_{\phi=0}^{\pi/4} \cos\phi\, d\phi \cdot [z]_0^1$$

$$\phi = 3\left[\sin\phi\right]_0^{\pi/4}[i]$$

$$\phi = 3\left[\sin\pi/4 - \sin 0°\right]$$

$$\boxed{\phi = 2.12 \ \omega b}$$

* Find the flux passing the portion of the plane $\phi = \frac{\pi}{4}$ Define by $0.01 \leq r \leq 0.05$ m and $0 \leq z \leq 2$ m. A current filament of 2.5 A is along z-axis in the $\overline{a_z}$ direction in the free Space.

$\boxed{\text{Free Space means } \mu = \mu_0}$

Given that,

$\phi = \pi/4$

$0.01 \leq r \ 0.05$ m

$0 \leq z \leq 2$ m.

$I = 2.5A \rightarrow$ z-axix.

Magnatic flux crosing cylindrical surface.

$$\phi = \int_s \overline{B}.ds$$

where $B = \mu H$

$$\overline{H} = \frac{I}{2\pi r} . \overline{a_\phi} \quad \text{(By amp circuital law)}$$

$$\overline{H} = \frac{2.5}{2\pi r}\overline{a_\phi} = \frac{1.25}{\pi r}\overline{a_\phi}$$

$$\overline{B} = \mu_0 H$$

$$\overline{B} = 4\pi \times 10^{-7} \times \frac{1.25}{\pi r}\overline{a_\phi}$$

$$\overline{B} = \frac{5\times10^{-7}}{r} \cdot \overline{a_\phi}$$

$\overline{dS}$ is normal to $\overline{a_\phi}$ in Cylindrical Co-ordinate System.

$$\overline{dS} = dr\,dz \cdot \overline{a_\phi}$$

$$\phi = \int_S \frac{5\times10^{-7}}{r} \cdot dr \cdot dz$$

$$\phi = \int_{r=0.01}^{0.05}\int_{z=0}^{2} \frac{5\times10^{-7}}{r}\, dr \cdot dz$$

$$\phi = 5\times10^{-7} \int_{r=0.01}^{0.05} \frac{1}{r} dr \int_{z=0}^{2} dz$$

$$\phi = 5\times10^{-7} \left[\ln r\right]_{0.01}^{0.05} \times \left[z\right]_0^2$$

$$\phi = 5\times10^{-7} \times 2 \ln\left[\frac{0.05}{0.01}\right]$$

$$\phi = 1.609\times10^{-6}\ Wb$$

$$\boxed{\phi = 1.609\ \mu Wb}$$

* Scalar & Vector Magnatic Potential

* In Electrostatics, Electrical potential 'V' Related with $\bar{E}$ as

P.V

$$\boxed{\bar{E} = -\nabla V}$$

* In magnetostatics → 2 Types of Potential
  (i) Scaler Magnatic potential (V_m)
  (ii) Vector magnatic potential ($\bar{A}$)

* To Study about Scaler & Vector magnatic potentials with help of two vector identition

[i] $\nabla \times \nabla V = 0$
[ii] $\nabla \cdot (\nabla \times \bar{A}) = 0$
} Vector Identition

Scalar magnetic Potential :-

Scalar magnetic potential has to Satisfy vector identity is.

$$\boxed{\nabla \times \nabla V_m = 0} \quad ---(i)$$

In magnatostatics, Vm is related $\bar{H}$ as

$$\boxed{\bar{H} = -\nabla V_m} \quad ---(ii)$$

From using equ(ii),

$$\nabla V_m = -\bar{H}$$

$$\nabla \times (-\bar{H}) = 0$$

$$\boxed{\nabla \times \bar{H} = 0} \quad ---(iii)$$

But $\boxed{\nabla \times \bar{H} \text{ (curl of } \bar{H}) = \bar{J}} \quad ---(iv)$

equ(iii) = equ(iv)

$\boxed{\bar{J}=0}$ (Surface free region)

$\therefore$ $\boxed{\nabla \times \bar{H} = 0}$ only for $\bar{J}=0$

Scaler magnatic potential V_m can be defined for Source free region [current density $\bar{J}=0$]

Laplace's equation for Scalar magnatic potential.

for closed surface

Apply divergence theorem.

$$\boxed{\int_S \bar{B}\cdot d\bar{S} = \int_V (\nabla\cdot\bar{B})\,dv = 0}$$

$\boxed{\nabla\cdot\bar{B} = 0}$ --- (i)

$\bar{B} = \mu\bar{H}$

$\nabla\cdot\mu\bar{H} = 0$

$\nabla\cdot\bar{H} = 0$ --- (ii) $(\mu \neq 0)$

In magnatostatics

$\bar{H} = -\nabla V_m$ --- (iii)

Sub (iii) in (ii)

$\nabla\cdot(-\nabla V_m) = 0$

$\boxed{\nabla^2 V_m = 0}$ Laplace equ for scalar magnatic potential.

* **Vector Magnetic Potential ($\bar{A}$)**

Vector magnetic potential has to Satisfy vector gndentity is $\boxed{\nabla\cdot(\nabla\times\bar{A}) = 0}$ ---(i)

For closed Surface,

Divergence of flux density is zero

is $\boxed{\nabla\cdot\bar{B} = 0}$ --(ii)

from equ (i) & (ii),

$\boxed{\bar{B} = \nabla\times\bar{A}}$ ---(iii)

But $\boxed{\nabla\times\bar{H} = \bar{J}}$ --- (iv)

~~But~~

$\bar{B} = \mu\bar{H}$; $\boxed{\bar{H} = \frac{\bar{B}}{\mu}}$ --- (v)

Sub (v) in (iv)

$\nabla\times\frac{\bar{B}}{\mu} = \bar{J}$; $\boxed{\nabla\times\bar{B} = \mu\bar{J}}$ --- (vi)

Sub (iii) in (vi)

$$\nabla\times\nabla\times\bar{A} = \mu\bar{J}$$

$$J = \frac{1}{\mu}\left[\nabla\times\nabla\times\bar{A}\right]$$

Where

$$\boxed{\nabla\times\nabla\times\bar{A} = \nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}}$$

$$\boxed{J = \frac{1}{\mu}\left[\nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}\right] = \bar{J}}$$ --- (vii)

* Poisson's Vector for Vector Magnetic potential.

from (vii)

$$\frac{1}{\mu}\left[\nabla(\nabla\cdot\bar{A}) - \nabla^2 A\right] = \bar{J}$$

If $\nabla\cdot\bar{A} = 0$ for complete defining of $\bar{A}$,

$$\frac{1}{\mu}\left[-\nabla^2 A\right] = \bar{J}$$

~~[illegible]~~ $\boxed{\nabla^2 A = -\mu\bar{J}} \rightarrow$ Poisson's equation for vector magnatic potential.

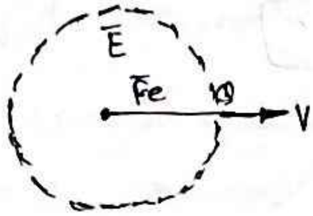
EEE

UNIT - IV Force in Magnetic fields

* force on a moving charge,

In electric fields,

$\bar{E} = \frac{\bar{F}}{Q}$



* force Exerted by $\bar{E}$ on moving charge Q is given by

$$\boxed{\bar{F_e} = Q\bar{E}} \quad ---(i)$$

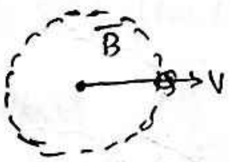
* $\bar{F_e}$ is independent on Velocity of moving charge 'V'

* $\bar{F_e}$ Performs work on moving charge 'Q' is,

$$\boxed{W = \bar{F}.\bar{dL} = F\,dL\cos\theta}$$

* In magnatic field

Consider that a charged is placed in a steady magnetic field. It experiences a force only if it is moving, Then.



* force $\bar{F_m}$ exerted by magnetic field $\bar{B}$ on moving charge with Velocity 'V' is given by.

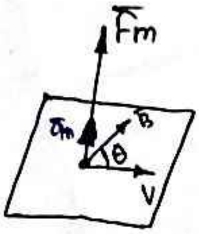
$$\boxed{\bar{F_m} = Q\bar{V}\times\bar{B}} \quad --- (ii)$$

Where
Q = Magnitude of charge, (C)
$\bar{V}$ = Velocity of charged Particle, (m/s).
$\bar{B}$ = Magnetic flux density (W/m²)

* Fm depends upon velocity of moving charge

* $\bar{F}_m$ is $\perp^r$ Containing V & B.

$\bar{F}_m$ can not perform work on a moving charge as it is at right angle to the direction of motion of charge



$$\boxed{\bar{F}\cdot d\bar{L} = 0}$$

* Fm is cannot perform work on moving charge Q.

is $W = F\cdot dl = F\,dl\,\cos\theta$

$\bar{F}_m$ is $\perp^r$ so, Assumed $\theta = 90°$

$$\boxed{W = F\,dl\,\cos 90'}$$

$$\boxed{W = 0}$$

* Total force on a moving charge Q in ~~electromagnetic fields is given by~~ the presence of both ~~fields~~ ($\bar{E}$ & $\bar{B}$) electric and magnetic field is given by.

$$\bar{F}_{Total} = \bar{F}_e + \bar{F}_m \quad (\bar{F}_{elect} + \bar{F}_{mag})$$

$$\bar{F} = Q\bar{E} + Q\bar{v}\times\bar{B}$$

$$\boxed{\bar{F} = Q\left[\bar{E} + (\bar{V}\times\bar{B})\right]}$$

Where,
Q = Magnitude of charged particle. (C)
$\bar{E}$ = Electric field intensity, (V/m)
$\bar{v}$ = Velocity of charged particle, (m/s)
$\bar{B}$ = Magnetic flux density (Wb/m²)

This is also called as Lorentz force equation;

Lorentz force equation relates mechanical force to the electrical force. If the mass of charge is 'm' then;

$$\bar{F} = m\bar{a}$$

$a = \frac{dv}{dt}$ $\boxed{\bar{F} = m\left[\frac{dv}{dt}\right] = Q\left[\bar{E} + (\bar{V}\times\bar{B})\right]}$ N

* A point charge of $Q = -1.2$ C has velocity $(\bar{V}) = (5\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z)$ m/s. find the magnitude of force excerted on the charge if

(a) $\bar{E} = (-18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z)$ V/m $= \boxed{\bar{F}_e = Q\bar{E}}$

(b) $\bar{B} = (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)$ T $= \boxed{\bar{F}_m = \bar{Q}\bar{v}\times\bar{B}}$

(c) Both are present simultaneously ($\bar{E}$ & $\bar{B}$)

$$\bar{F} = Q\left[\bar{E} + (\bar{V}\times\bar{B})\right]$$

=> Given data

$Q = -1.2$ C

$V = (5\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z)$ m/s

$F = ?$

find force

(i) $\bar{F}_e = Q\bar{E} = -1.2(-18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z)$

$\bar{F}_e = 21.6\,\bar{a}_x - 6\,\bar{a}_y + 12\bar{a}_z$

$\bar{F}_e = \sqrt{(21.6)^2 + (-6)^2 + (12)^2} = 25.42$ N

(ii) find $\bar{F}_m$

$\bar{F}_m = \bar{Q}\bar{v}\times\bar{B}$

$-1.2(5\bar{a}_x + 2\bar{a}_y - 3\bar{a}_z) \times (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)$

$= (-6\bar{a}_x - 2.4\bar{a}_y + 3.6\bar{a}_z) \times (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)$

$$\bar{F}_m = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ -6 & -2.4 & 3.6 \\ -4 & 4 & 3 \end{vmatrix} = \bar{a}_x\left[-7.2 - 14.4\right] - \bar{a}_y\left[-18 + 14.4\right] + \bar{a}_z\left[-24 - 9.6\right]$$

$$\bar{F}_m = -21.6\,\bar{a}_x + 3.6\,\bar{a}_y - 33.6\,\bar{a}_z$$

$$F_m = \sqrt{(-21.6)^2 + (3.6)^2 + (-33.6)^2}$$

$$F_m = 40.1058\ N$$

(iii) The total force excerted by both the fields ($\bar{E}$ and $\bar{B}$) on a charge is given by.

$$\bar{F} = \bar{F}_e + \bar{F}_m = Q\bar{E} + Q\bar{v}\times\bar{B}$$

$$\Rightarrow \left[21.6\bar{a}_x - 6\bar{a}_y + 12\bar{a}_z + (-21.6\bar{a}_x + 3.6\bar{a}_y - 33.6\bar{a}_z)\right]$$

$$\bar{F} = (0\bar{a}_x - 2.4\bar{a}_y - 21.6\bar{a}_z)\ N$$

Thus the magnitude of the total force exerted is given by,

$$|\bar{F}| = \sqrt{(0)^2 + (-2.4)^2 + (-21.6)^2} = 21.7329\ N \quad \underline{\text{Ans}}$$

* Force on a differential current Element :-

⇒ Force excerted on differential element of charge dQ moving with velocity V in magnatic field $\bar{B}$ is given by.

$$\boxed{d\bar{F} = dQ\,v \times \bar{B}}\ N \quad --- (i)$$

dQ is differential charge can be expressed interms of volume charge density is given by.

$$\boxed{dQ = \rho_v dv} \quad —— (ii)$$

Sub dQ from (ii) in equ (i),

$$\boxed{d\bar{F} = \rho_v dv\, V \times \bar{B}} \quad --- (iii)$$

* Current density $\bar{J}$ can be expressed interms of velocity of volume charge density is

$$\boxed{\bar{J} = \rho_v V} \quad —— (iv)$$

At the place of $\rho_v v$ Substitute J in equ (iii),

$$\boxed{\bar{dF} = J \times \bar{B} \cdot dv} \; --- (iv) \quad [\text{From (iv) } \rho_v v = J]$$

* Current elements are related as.

$$\boxed{\bar{J}dv = \bar{K} \cdot ds = \bar{I} \cdot dL} \text{ --- (vi)}$$

At the place of $\bar{J}dv$ Sub $\bar{K} \cdot dS$, & $\bar{I} \cdot dL$ in equ (v)(iii),

$$\boxed{\bar{dF} = \bar{K} \times \bar{B} \cdot ds} \quad \text{from (vi)}$$

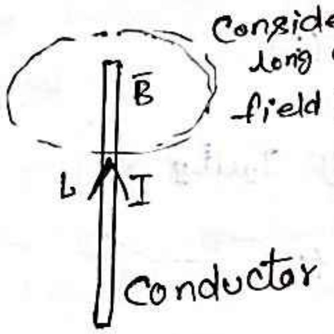
Where,

K => Surface current density,

(or) Sub the Value of $\bar{J}dv$ at the place $\bar{I} \cdot dL$

* $$\boxed{\bar{dF} = I\bar{dL} \times \bar{B}} \quad (\text{From (vi)})$$

* Force on Straight & Long Current Carrying Conductor.



Consider a Conductor is straight long carrying current I and the field $\bar{B}$ is uniform along it,

$$\boxed{\bar{A} \times \bar{B} = |A||B| \sin\theta_{\cdot}}$$

Force excerted on current carraying Conductor is given by.

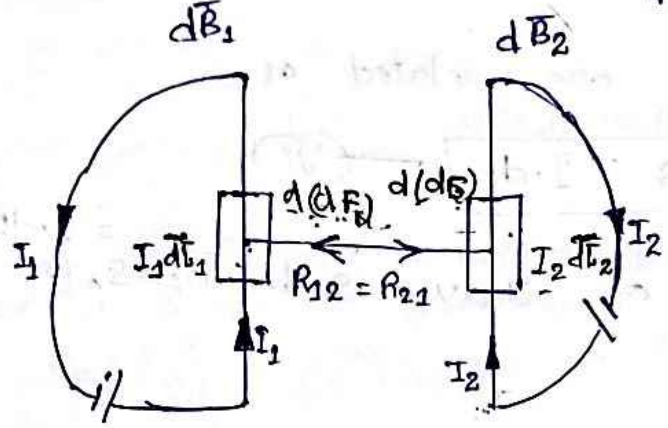
$$\cancel{\bar{F} = I\bar{L}} \times B \quad \boxed{\bar{F} = I\bar{L} \times \bar{B}}$$

* Magnitude of force is given by.

$$\boxed{F = BIL \sin\theta}$$

* Force between Two differential Current Elements

Elements :- Let us now consider two current elements $I_1 d\bar{L}_1$ and $I_2 d\bar{L}_2$ as shown in fig. And the direction of I_1 & I_2 are same.



$d(dF_1)$ or $d(dF_2)$ => force between two differential current elements.

* Force $d(dF_1)$ excerted on current element $I_1 d\bar{L}_1$ due to $d\bar{B}_2$ produced by other current element $I_2 d\bar{L}_2$

$$\boxed{d(d\bar{F}_1) = I_1 d\bar{L}_1 \times d\bar{B}_2} \quad \text{---- (i)}$$

* $d\bar{B}_2$ is field produced by $I_2 d\bar{L}_2$

$$\boxed{d\bar{B}_2 = \mu d\bar{H}_2} \quad \text{---- (ii)}$$

According to Biot - Savart's law

$$\boxed{d\bar{H}_2 = \frac{\left[(I_2 d\bar{L}_2) \times \bar{a}_{R_{21}}\right]}{4\pi R_{21}^2}} \quad \text{---- (iii)}$$

Sub equ(ii) & (iii) in equ(i),

$$\boxed{d(d\bar{F}_1) = \frac{I_1 d\bar{L}_1 \times \mu (I_2 d\bar{L}_2 \times \bar{a}_{R_{21}})}{4\pi R_{21}^2}}$$

* Total force F_1 is obtained,

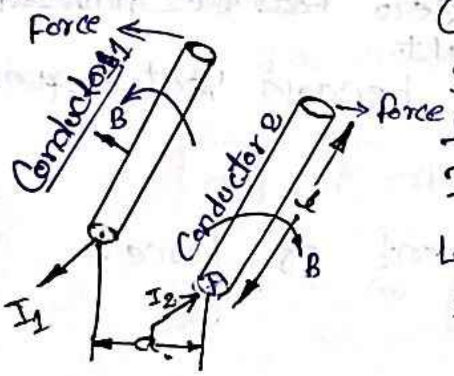
$$\boxed{F_1 = \frac{\mu I_1 I_2}{4\pi R_{21}^2} \oint_{L_1} d\bar{L}_1 \oint_{L_2} (d\bar{L}_2 \times \bar{a}_{R_{21}})}$$

III'y

$$\boxed{F_2 = \frac{\mu I_1 I_2}{4\pi R_{12}^2} \oint_{L_2} d\bar{L}_2 \oint_{L_1} (d\bar{L}_1 \times \bar{a}_{R_{12}})}$$

* Thus above condition also indicates that both the forces F_1 and F_2 obey Newton's third law that for every action their is equal and opposite Reaction.

* Force between Straight & long two parallel current carrying conductors



Consider two parallel long straight conductors of length l each carrying currents I_1 and I_2 as shown in fig. Let d be the distance of speration between the two conductors.

* Force between two parallel conductor carrying current is given by,

$$\boxed{\bar{F} = \frac{\mu I_1 I_2 l}{2\pi d}} \; N$$

Note:- That if the directions of the ~~electron~~ currents through the conductors are same, then two conductors attract each other. while if the directions of the currents through the conductors are opposite, then two conductors repel each other.

* Magnetic Torque ($\bar{T}$)

* Force Exerted on differential current element is given by,

$$\boxed{d\bar{F} = I d\bar{L} \times \bar{B}}$$

Total force on current element is.

$$\boxed{F = \oint d\bar{F} = \oint I d\bar{L} \times \bar{B}}$$

$$\bar{F} = I\oint d\bar{L} \times \bar{B}$$

$$[\bar{A}\times\bar{B} = -(\bar{B}\times\bar{A})]$$

$$\bar{F} = I\oint -(\bar{B}\times d\bar{L})$$

$$\bar{F} = -I \cdot B \times \oint d\bar{L}$$

But we know that for a closed path, $d\bar{L} = 0$;

$$\boxed{\oint d\bar{L} = 0}$$

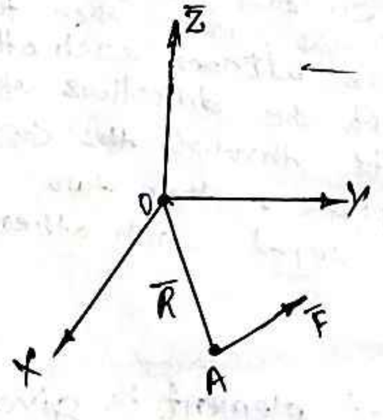
then $\bar{F} = 0$

Let us consider a new, ~~But~~ Vector quantity which is not equal to zero ~~because have unit~~ even though $\bar{F} = 0$; because Vector quantity have unit vector.

Magnetic Torque or moment of force;

$$\boxed{\bar{T} = \bar{R}\times\bar{F}} \text{ N-m}$$

$\bar{R}$ — Moment of Arm



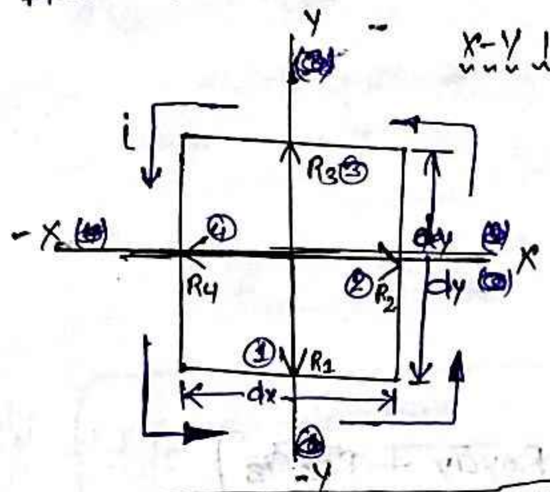
Consider a point A at which force $\bar{F}$ is applied as shown in fig. Let $\bar{R}$ be the arm of from origin O at point A. Then the torque $\bar{T}$ about the origin is nothing but a vector product of $\bar{R}$ & $\bar{F}$.

$\bar{T}$ is normal to $\bar{R}$ & $\bar{F}$.

$$\boxed{\bar{T} = \bar{R}\times\bar{F}} \text{ N-m}$$

* Torque on Current loop placed in a Magnetic field :-

Consider a differential current loop placed in x-y ~~Let us consider a x-y plane~~ plane in the magnetic field $\bar{B}$. The loop is placed in the plane such that the sides of the loop are parallel to the axes respectively. Let dx and dy be the lengths of the sides of the loop as shown in fig.



$$\boxed{\text{At origin } B = B_0 = B_{0x}\bar{a}_x + B_{0y}\bar{a}_y + B_{0z}\bar{a}_z} \quad \text{-- (i)}$$

*** Torque on side (1) of current loop, Side (1) is in (-) side of y axix**

$$\boxed{dT_1 = \bar{R}_1 \times d\bar{F}_1} \quad \text{—— (ii)}$$

* $\bar{R}_1 = -\frac{dy}{2} \cdot \bar{a}_y$ ((-) side of y-axix)

* $d\bar{F}_1 = I d\bar{L}_1 \times \bar{B}_0$

* $d\bar{L}_1 = dx \cdot \bar{a}_x$

* $\bar{B}_0 = B_{0x}\bar{a}_x + B_{0y}\bar{a}_y + B_{0z}\bar{a}_z$

$$dF_1 = I dx\, \bar{a}_x \times \left[B_{0x}\bar{a}_x + B_{0y}\bar{a}_y + B_{0z}\bar{a}_z\right]$$

$$\boxed{\bar{a}_x \cdot \bar{a}_x = 0 \; ; \; \bar{a}_x \times \bar{a}_y = \bar{a}_z \; ; \; \bar{a}_y \times \bar{a}_z = \bar{a}_x}$$

$$\boxed{dF_1 = I dx \left[B_{0y}\bar{a}_z - B_{0z}\bar{a}_y\right]}$$

$$\boxed{dT_1 = -\frac{dy}{2} \overset{\bar{a}_y}{\cdot} I dx \left[B_{0y}\bar{a}_z - B_{0z}\bar{a}_y\right]} \quad \text{(iii)}$$

$$\boxed{dT_1 = -\frac{I dx dy}{2} \left[B_{0y}\bar{a}_x - 0\right]} \quad \text{--- (iii)}$$

* Torque on Side (3) of current loop;

$$d\bar{T}_3 = R_3 \times d\bar{F}_3$$

$$* \ \bar{R}_3 = \frac{dy}{2} \cdot \bar{a}_y$$

$$d\bar{F}_3 = I d\bar{L}_3 \times \bar{B}_0$$

$$* \ d\bar{L}_3 = -dx \cdot \bar{a}_x$$

$$d\bar{F}_3 = -I dx \bar{a}_x \left[B_{0x}\bar{a}_x + B_{0y}\bar{a}_y + B_{0z}\bar{a}_z \right]$$

$$d\bar{F}_3 = -I dx \left[B_{0y}\bar{a}_z - B_{0z}\bar{a}_y \right]$$

$$\boxed{d\bar{T}_3 = -\frac{dy}{2} \bar{a}_y \, I dx \left(B_{0y}\bar{a}_z - B_{0z}\bar{a}_y \right)} \quad --- \text{(iv)}$$

$$d\bar{T}_3 = \frac{-I dx dy}{2} B_{0y} \cdot \bar{a}_x$$

$$\boxed{\left(d\bar{T}_1 + d\bar{T}_3 \right) = -I dx dy \, B_{0y}\bar{a}_x} \quad --- \text{(V)}$$

* Torque $d\bar{T}_2$ on side ② of differential of current loop

$$d\bar{T}_2 = R_2 \times d\bar{F}_2$$

$$* \ R_2 = \frac{dx}{2} \cdot \bar{a}_x$$

$$* \ d\bar{F}_2 = I d\bar{L}_2 \times \bar{B}_0$$

$$d\bar{L}_2 = dy \cdot \bar{a}_y$$

* $I d\bar{L}_2 = I dy . \bar{a}_y$.

$$d\bar{F}_2 = I dy . \bar{a}_y \times [B_{ox}\bar{a}_x + B_{oy}\bar{a}_y + B_{oz}\bar{a}_z]$$

$$d\bar{T}_2 = \frac{dx}{2} \bar{a}_x \, I dy \times [-B_{ox}\bar{a}_z + B_{oz}\bar{a}_x]$$

$$d\bar{T}_2 = \frac{I dx dy}{2} \bar{a}_x \times [-B_{ox}\bar{a}_z + B_{oz}\bar{a}_x]$$

$|||^{ly}$

$$\boxed{d\bar{T}_2 = \frac{I dx dy}{2} B_{ox}\bar{a}_y} \text{ --- (vi)}$$

Torque $d\bar{T}_4$ on side (4) in same on torque on side ②

$$\boxed{d\bar{T}_4 = d\bar{T}_2 = \frac{I dx dy}{2} B_{ox}\bar{a}_y} \text{ --- (vii)}$$

$$\boxed{d\bar{T}_2 + d\bar{T}_4 = I dx dy \; B_{ox}\bar{a}_y} \text{ --- (viii)}$$

Total torque on differential current loop, is given by,

$$d\bar{T} = d\bar{T}_1 + d\bar{T}_2 + d\bar{T}_3 + d\bar{T}_4$$

$$d\bar{T} = (d\bar{T}_1 + d\bar{T}_3) + (d\bar{T}_2 + d\bar{T}_4)$$

$$d\bar{T} = -I dx dy \, B_{oy}\bar{a}_x + I dx dy \, B_{ox}\bar{a}_y$$

$$d\bar{T} = I dx dy \, [B_{ox}\bar{a}_y - B_{oy}.\bar{a}_x]$$

Where

$\bar{a}_z \times \bar{B}_o$

$= \bar{a}_z \times [B_{ox}\bar{a}_x + B_{oy}\bar{a}_y + B_{oz}\bar{a}_z]$

$$\boxed{a_z \times \bar{B}_o = B_{ox}\bar{a}_y - B_{oy}\bar{a}_x}$$

$$d\bar{T} = I\,dx\,dy \cdot \bar{a}_z \times \bar{B}_0$$

$$\bar{B}_0 = \bar{B}$$

$$dT = I\,dx\,dy \cdot \bar{a}_z \times \bar{B}$$

* $d\bar{S} =$ ~~dxdydz~~ $dx\,dy \cdot \bar{a}_z$ = Vector Area of differential current loop

* $$\boxed{d\bar{T} = I\,d\bar{S} \times \bar{B}}$$

* Magnetic Diople Moment $[\bar{m}]$

=> The magnetic dipole moment of a current loop is defined as product of current through the loop and the area of the loop, directed normal to the current loop.

* It is denoted by $(\bar{m})$. and magnetic diople moment $(\bar{m})$ is given by.

$$\boxed{\bar{m} = IS \cdot \bar{a}_n} \; Am^2$$

Where,
$\bar{a}_n$ is unit vector normal to current loop

$$\bar{a}_n = \bar{a}_z$$

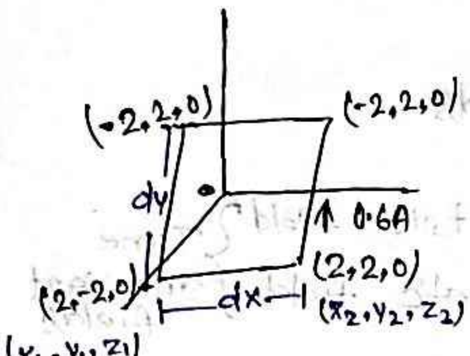
* for dipole moment $(\bar{m})$
Torque on current loop is given by,

$$\boxed{T = \bar{M} \times \bar{B}} \; N\text{-}m,$$

Calculate the Vector torque on a square root as show in the fig. about an origin at A in the magnetic field $\bar{B}$ given origin at.

(a) $A(0,0,0)$ & $\bar{B} = 100\bar{a}_y$ mT

(b) $A(0,0,0)$ & $\bar{B} = 200\bar{a}_x + 100\bar{a}_y$ mT



* find vector torque on square current loop.

* Torque on current loop is given by,

$$\boxed{\bar{T} = I\bar{S} \times \bar{B}}$$

$$\bar{S} = dx \cdot dy \cdot \bar{a}_z$$

$$dx = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$dx = \sqrt{(2-2)^2 + (2+2)^4 + (0-0)^2} = 4\bar{a}_x$$

* $$dy = \sqrt{(-2-2)^2 + (-2+2)^2 + (0-0)^2} = 4$$

$$dy = 4\bar{a}_y$$

$$S = 4\bar{a}_x \cdot 4\bar{a}_y \cdot \bar{a}_z$$

$$S = 16\bar{a}_x \cdot \bar{a}_y$$

$$S = 16\bar{a}_z$$

* $$T = 0.6 \times 16\bar{a}_z \times [100\bar{a}_y] \times 10^{-3}$$

$$T = -960\bar{a}_z \cdot \bar{a}_y \times 10^{-3}$$

$$\boxed{T = -0.96\,\bar{a}_x} \text{ N/m}$$

(b) $\tau = 0.16 \times 16\bar{a}_z \times [200\bar{a}_x + 100\bar{a}_y]$

$= [1920\,\bar{a}_y - 960\,\bar{a}_x]\, 10^{-3}$

$\boxed{\tau = 1.92\,\bar{a}_y - 0.96\,\bar{a}_x}$ N-m

UNIT-6

Time Varying fields

* Electrostatics → Static electric field
* Magneto statics → Static magnetic field

} Time invariant fields

* Time Variant fields,

Electro magnetic fields → Time Varying field

Electric field
Magnetic field
} interdependent

Faraday law of electromagnetic induction:-

According Faraday's

Induced EMF in Conductor is given by.

$\boxed{e \propto \frac{d\psi}{dt}}$ $\frac{d\psi}{dt}$ = Rate of change of flux linkage

ψ = Total flux linkage

$\psi = N\phi$

$e \propto \frac{d}{dt}(N\phi)$

$$\boxed{e = \frac{-N\,d\phi}{dt}}$$

for $N = 1$

$$\boxed{e = \frac{-d\phi}{dt}} \text{ V} \text{ — (1)}$$

Magnetic flux is given by,

$$\boxed{\phi = \int_S \bar{B} \cdot d\bar{S}} \quad \text{--- (ii)}$$

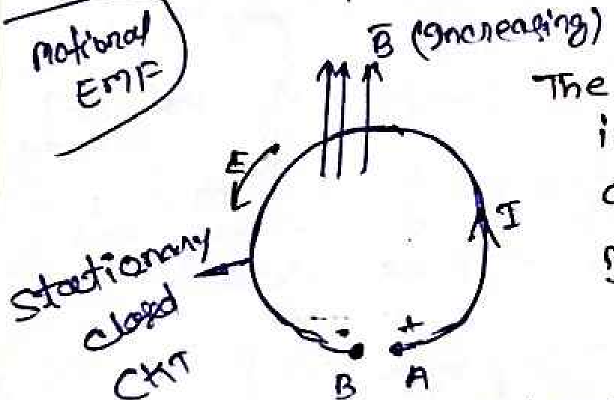
Sub equ (ii) in equ (i),

$$e = \frac{-d}{dt}\left[\int_S \bar{B} \cdot d\bar{S}\right]$$

$$e = -$$

* Statically Induced EMF due to a Stationary closed path (CKT) in time Varying $\bar{B}$ field.

motional EMF



The condition (or transformer emf) in which a closed path is Stationary and $\bar{B}$ field is varying with respect to time is as shown in fig.

EMF Induced → Statically Induced EMF

EMF Induced in closed path (CKT) is given by,

$$\boxed{\oint \bar{E}\cdot d\bar{l} = -\frac{d}{dt}\int \bar{B}\cdot d\bar{S}}$$

Where,

$\bar{B} \rightarrow$ Time Varying field

$$\boxed{\oint \bar{E}\cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{S}} \quad \text{---(i)}$$

Statical induced EMF (or) This is similar to Transformer EMF.

By apply Stokes theorem

Line integral converted into surface integral.

$$\boxed{\oint_L \bar{E}\cdot d\bar{L} = \int_S (\nabla\times\bar{E})\, d\bar{S}} \quad \text{---(ii)}$$

equ (ii) = (i)

$$\int_S (\nabla\times\bar{E})\, d\bar{S} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{S}$$

$$\boxed{\nabla\times\bar{E} = \frac{-\partial \bar{B}}{\partial t}} \quad \text{---(iii)}$$

For static magnetic field ($\bar{B}$ = Constent),

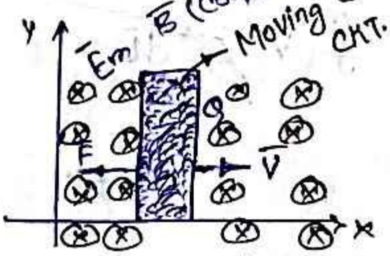
$$* \ \boxed{\nabla\times\bar{E} = 0}$$

$$* \ \boxed{\oint \bar{E}\cdot d\bar{L} = 0}$$

* Dynamically induced EMF due to a moving closed path (CKT) in static $\bar{B}$ field.

Static EMF

(Motional emf)



The condition ~~method~~ mentioned above is represented in the fig.

* Consider charge 'Q' moving with velocity 'V' in a magnetic field $\bar{B}$ then force of moving charge 'Q' is given by.

$$\boxed{\bar{F} = Q\bar{V} \times \bar{B}} \quad \text{--- (i)}$$

* $\bar{E}_m$ electric field intensity is force per unit charge.

$$\bar{E}_m = \frac{\bar{F}}{Q}$$

$$\bar{E}_m = \frac{\cancel{Q}\bar{V} \times \bar{B}}{\cancel{Q}}$$

$$\boxed{\bar{E}_m = \bar{V} \times \bar{B}} \quad \text{--- (ii)}$$

* According to faraday's law in time varying fields EMF induced in closed path (CKT) is given by,

$$\boxed{\oint \bar{E}_m d\bar{L} = \oint (\bar{V} \times \bar{B}) \cdot d\bar{L}} \quad \text{--- (iii)}$$ Dynamically induced EMF

* Moving closed path (CKT) in time varying $\bar{B}$ field.

* Total EMF induced = Statically induced EMF + (Transformer EMF)

Dynamical induced EMF (Motional induced EMF)

$$\oint \bar{E} \cdot d\bar{L} = -\int_S \frac{\partial B}{\partial t} \cdot d\bar{S} + \oint (\bar{V} \times \bar{B})\, d\bar{L}$$

A Conductor 1 c.m in length is parallel to z-axis and rotates a 1200 rPM and having reduce of 25 cm find induced voltage if the radial field is given by. $\bar{B} = 0.5\bar{a}_r$ T.

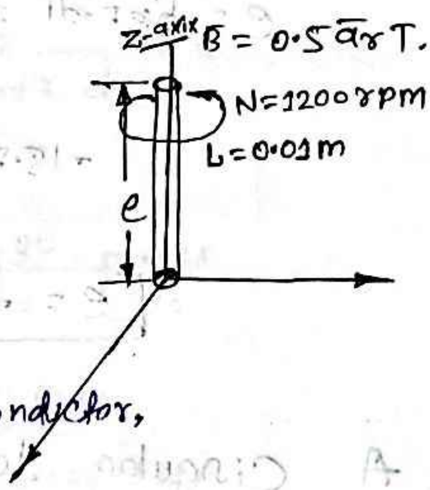
Given data

=> $L = 1$ c.m $= 0.01$ m

$r = 25$ c.m $= 0.25$ m

$N = 1200$ rPM

$\bar{B} = 0.5\bar{a}_r T$



Find EMF induced in Conductor,

$$\oint \bar{E}.d\bar{L} = \oint(\bar{V}\times\bar{B}).d\bar{L}$$

$V = ?$ m/s

$N = 1200$ rev/min

$$N = \frac{1200}{60} \text{ rev/sec} = 20 \text{ rev/sec.}$$

for 1 rev distance travelled $= 2\pi r$

for 20 rev " " $= 20\times2\pi r = 40\pi r$ m/sec

$V = 40\pi r$ m/sec in 'ϕ' direction.

$$\bar{V} = 40\pi r.\bar{a}_\phi$$

$d\bar{L} = dz.\bar{a}_z$

$$\oint E.d\bar{L} = \int(40\pi r.\bar{a}_\phi)\times(0.5\,\bar{a}_r).dz.\bar{a}_z$$

$$\oint E.d\bar{L} = 40\times\pi\times0.5\times0.25\,[\bar{a}_\phi\times\bar{a}_r]\int dz.\bar{a}_z$$

$$= 15.70\,(-\bar{a}_z)\int dz.\bar{a}_z$$

$$= -15.7 \int_{z=0}^{0.01} dz$$

$$e = \oint \bar{E} \cdot d\bar{l} = -15.7 [z]_0^{0.01}$$

$$-15.7 \times 0.01 \qquad e = -0.157$$

$$\boxed{e = -157\, mV}$$

A A Circular loop Conductor lies in a plane $Z = 0$ and has a radius of 0.1m and Resistance of 5Ω, given $\bar{B} = 0.2 \sin 10^3 t$ Determine current in the loop.

$\Rightarrow$ $Z = 0$ (||el to X Y plane)

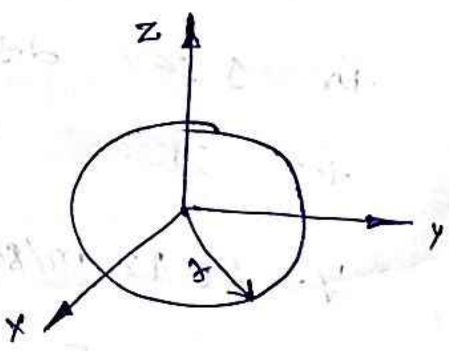
$r = 0.1\ m$

$\bar{B} = 0.2 \sin 10^3 t \cdot \bar{a}_z$

$R = 5\Omega$

Find (i) = ?

$i = \dfrac{e}{R}$ (EMF induced)



Find EMF induced in Circular loop.

$$e = \frac{-d\phi}{dt}$$

$$\phi = \int_S \bar{B} \cdot d\bar{S}$$

in Cylindrical Co-ordinate,

$$d\bar{S} = r\,dr.d\phi\,\bar{a}_z$$

$$\phi = \int_{r=0}^{0.1}\int_{\phi=0}^{2\pi} 0.25 \sin 10^3 t \cdot \bar{a}_z \cdot r\,dr\,d\phi\,\bar{a}_z$$

$$\phi = 0.02 \sin 10^3 t \int_{r=0}^{0.1}\int_{\phi=0}^{2\pi} r\,dr \cdot d\phi$$

$$\phi = 0.02 \sin 10^3 t \left[\frac{r^2}{2}\right]_0^{0.1} \left[\phi\right]_0^{2\pi}$$

$$\phi = 0.02 \sin 10^3 t \left(\frac{0.1}{2}\right)(2\pi)$$

$$\phi = 0.2 \sin 10^3 t \,\cancel{(\;)}\,(0.005)(2\pi)$$

$$e = -\cancel{\frac{d}{dt}(0.012 \sin 10^3 t)} \quad 6.28 \times 10^{-3} \sin(10^3 t)$$

$$e = -\frac{d}{dt}\left[6.28 \times 10^{-3} \sin 10^3 t\right]$$

$$\cancel{e = -0.012 \times \cos 10^3 t \times 10^3}$$

$$\boxed{\cancel{e = -12 \cos 10^3 t}} \checkmark \quad e = -6.28 \times 10^{-3} \times \cos 10^3 t \times 10^3$$

$$\boxed{e = -6.28 \cos 10^3 t}$$

$$\cancel{i = \frac{e}{R} = \frac{-12 \cos 10^3 t}{5}}$$

$$i = \frac{e}{R} = \frac{-6.28 \cos 10^3 t}{5}$$

$$\boxed{\cancel{i = -2.4 \cos 10^3 t}} \text{ A}$$

$$\boxed{i = 1.256 \cos 10^3 t} \text{ A}$$

* Modified Amper's circuital law of for time varying fields:-

* Amperes Circutal law for a static field is given by

$$\boxed{\nabla \times \bar{H} = \bar{J}} \quad \text{---} \quad (i)$$

Taking divergence on both sides

$$\boxed{\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J}}$$

According to vector identity.
Divergence of curl of any vector is zero

i.e $(\nabla\cdot(\nabla\times\bar{H})) = 0$

$$\boxed{\nabla\cdot\bar{J} = 0} \text{ --- (ii)}$$

According to equ of Continuity

$$\boxed{\nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t}} \text{ --- (iii)}$$

$\frac{\partial \rho_v}{\partial t} = 0 \longrightarrow$ for Static fields.

* Modify Amphere's circuital law for time varying fields,

By adding $\bar{N}$

$$\boxed{\nabla\times\bar{H} = \bar{J}+\bar{N}} \text{ --- (iv)}$$

Again taking divergence on b.side.

$$\boxed{\nabla\cdot(\nabla\times\bar{H}) = \nabla\cdot\bar{J}+\nabla\cdot\bar{N}} \text{ --- (V)}$$

* $\nabla(\nabla\times\bar{H}) = 0$,

$$\nabla\cdot\bar{J} + \nabla\cdot\bar{N} = 0$$

* $\nabla\cdot\bar{J} = -\frac{\partial\rho_v}{\partial t}$

$$-\frac{\partial\rho_v}{\partial t} + \nabla\cdot\bar{N} = 0$$

$$\boxed{\nabla\cdot\bar{N} = \frac{\partial\rho_v}{\partial t}} \text{ --- (6)}$$

According to Gauss law.

$$\rho_v = \nabla\cdot\bar{D}$$

$$\nabla\cdot\bar{N} = \frac{\partial}{\partial t}(\nabla\cdot\bar{D})$$

$$\boxed{\nabla \cdot \bar{N} = \nabla \cdot \frac{\partial D}{\partial t}} \quad --(7)$$

$$\boxed{\bar{N} = \frac{\partial \bar{D}}{\partial t}} \quad --(8)$$

Sub equ (8) in equ (4).

$$\nabla \times \bar{H} = \bar{J} + \bar{N}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial D}{\partial t}$$

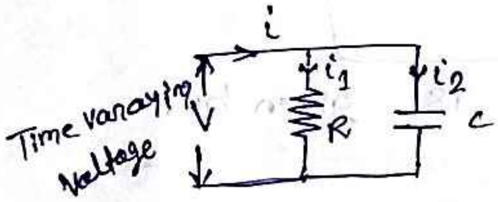
$J = J_c =$ Conduction Current density.

$J_D = \frac{\partial D}{\partial t} \Rightarrow$ Displacement current density.

$$\boxed{\nabla \times \bar{H} = \bar{J_c} + \bar{J_D}}$$

* Displacement Current & Displacement Current density..

=> Consider parallel RC circuit.



* Current through Resistance (R) is given by,

$$i_1 = \frac{V}{R} = i_c \Rightarrow \text{Conduction Current}$$

* A is area of cross section of Resistor 'R'

* Conduction current density is given by.

$$\boxed{\bar{J_c} = \frac{i_c}{A} = \sigma \bar{E}}$$

* Current through i_{∞}= Capacitor is given by.

$$\boxed{i_2 = C\frac{dv}{dt}}$$

$i_2 = i_D \longrightarrow$ Displacement current.

* 'A' Area of cross section of each parallel plate.

$$\bar{J}_D = \frac{i_D}{A}$$

$$\bar{J}_D = \frac{C}{A} \frac{dV}{dt} \qquad C = \frac{\epsilon A}{d}$$

$$J_D = \frac{\epsilon A\, dV}{d A\, dt} \qquad J_D = \frac{\epsilon}{d} \frac{dV}{dt}$$

* E due to Voltage between two plates;

$$\boxed{E = \frac{V}{d}} \quad \& \quad V = Ed.$$

$$\bar{J}_D = \frac{\epsilon}{d} \frac{d}{dt}(Ed)$$

$$\boxed{\bar{J}_D = \epsilon \frac{dE}{dt} = j\epsilon\omega \bar{E}}$$

* Total current density is given by,

$$\boxed{\bar{J} = \bar{J}_c + \bar{J}_D}$$

$$\boxed{\bar{J} = \sigma \bar{E} + j\epsilon\omega \bar{E}}$$

$$\boxed{\bar{J} = \sigma \bar{E} + j\omega\epsilon \bar{E}}$$

Ratio between magnitude $\bar{J}_c$ & $\bar{J}_D$

$$\frac{|\bar{J}_c|}{} = \frac{|\sigma \bar{E}|}{|j\omega\epsilon \bar{E}|}, \quad \boxed{\bar{D} = \epsilon \bar{E}}$$

$$\frac{|\bar{J}_c|}{|\bar{J}_D|} = \frac{\sigma}{\omega\epsilon}$$

* In a given dielectric medium, Conduction density $J_c = 0.02 \sin 10^9 t$ A/m². find the displacement current density if $\sigma = 10^3$ S/m find $\varepsilon_r = 6.5$

Given data,

$\Rightarrow$

$J_c = 0.02 \sin 10^9 t \ A/m^2 = \sin 10^9 t$

$\sigma = 10^3 S/m$

$\varepsilon_r = 6.5$

$\omega = 10^9$

$J_D = ?$

& find $\bar{J}_D$

$$\frac{\bar{J_c}}{|J_d|} = \frac{\sigma E}{\omega}$$

* General field Relationship Varying electric & magnetic field.

$\Rightarrow$ From faraday's law.

$\bar{E}$ & $\bar{B}$ are related as,

$$\boxed{\nabla \times E = -\frac{\partial \bar{B}}{\partial t}} \quad \text{--- (i)}$$

$$\boxed{\bar{B} = \nabla \times A} \quad \text{--- (ii)}$$

Where $\bar{A}$ = Vector magnetic potential.

Sub (ii) in (i),

$$\nabla \times E = \frac{-\partial}{\partial t}(\nabla \times \bar{A})$$

$$\boxed{\nabla \times \bar{E} = -\nabla \times \frac{\partial A}{\partial t}} \quad \text{---(iii)}$$

$$\nabla \times \bar{E} + \nabla \times \frac{\partial \bar{A}}{\partial t} = 0$$

$$\boxed{\nabla \times \left(\bar{E} + \frac{\partial \bar{A}}{\partial t}\right) = 0} \quad \text{--- (iv)}$$

* From vector identity curl of gradient of scaler is zero.

$$\boxed{\nabla \times [-\nabla \cdot V] = 0} \quad \text{--- (v)}$$

$$\nabla \times \left[\bar{E} + \frac{\partial \bar{A}}{\partial t}\right] = \nabla \times (-\nabla V)$$

$$\bar{E} + \frac{\partial \bar{A}}{\partial t} = -\nabla V$$

$$\boxed{\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t}} \quad \text{---- (vi)} \quad \text{(for time varying field)}$$

* For static field $\bar{A}$ = Constant.

$$\boxed{\bar{E} = -\nabla V} \quad \text{— (for Static field)}$$

* for closed Surface :-

'I' is current flows out of Surface is given by,

$$I = \frac{dq}{dt}$$

Let $Q = -q$, (flows in word of surface),

$$\boxed{I = \frac{dq}{dt}} \quad \text{(vii)}$$

* Q is charge in terms of volume charge density is given by,

$$\boxed{Q = \int_v \rho_v dv} \quad — (8)$$

Sub equ (8) in (7)

$$I = -\frac{d}{dt}\left(\int \rho_v dv\right)$$

$$\boxed{I = -\int_v \frac{d\rho_v}{dt} \cdot dv} \quad --- (9)$$

from Gauss law,

$$\boxed{\rho_v = \nabla \cdot \bar{D}} \quad — (10)$$

$$I = -\int_v \frac{d}{dt}(\nabla \cdot \bar{D}) dv$$

$$\boxed{I = -\int_v \nabla \cdot \frac{d\bar{D}}{dt} \cdot dv} \quad --- (11)$$

In terms current density as,

$$\boxed{I = \int_v \bar{J} \cdot d\bar{S}}$$

* Apply Divergence theorem converts surface integrate into volume integral.

$$\int_s J \cdot ds = \int_v \nabla \cdot \bar{J} dv$$

$$\therefore \int_v \nabla \cdot \bar{J} dv = -\int_v \nabla \cdot \frac{d\bar{D}}{dt} dv$$

$$\boxed{\nabla \cdot \bar{J} = -\frac{d \cdot \bar{D}}{dt}}$$

Equation of continuity of current in point form (or) in differential form.

* Maxwell's Equation for time varying Electric & magnetic field in point (differential) form or integral form.

=> 4 Equation

[i] Ampere's CKTal law

[ii] faraday's law

[iii] Gauss's law $\rightarrow$ for E

$\rightarrow$ for M

} 4 equation in point form & integral form.

Maxwell's equations:-

[1] From Amper's CKTal law

$$\boxed{\oint_L \bar{H} \cdot dL = I_{enc}} \quad \text{--- (i)}$$

I enclosed interms of current density in

$$\boxed{I_{enc} = \int_S \bar{J} \cdot dS} \quad \text{--- (ii)}$$

For time varaying fields, Add $\left(I_D = \int_S \frac{\partial \bar{D}}{\partial t} \cdot d\bar{S}\right)$ to equ ②

$$\boxed{I_{enc} = \int_S \bar{J} \cdot dS + \int_S \frac{\partial \bar{D}}{\partial t} \cdot d\bar{S}} \quad \text{--- (iii)}$$

Sub equ (iii) in equ(i),

$$\oint_L \bar{H} \cdot d\bar{L} = \int_S \bar{J} \cdot dS + \int_S \frac{\partial \bar{D}}{\partial t} \cdot d\bar{S}$$

$$\boxed{\oint_L \bar{H} \cdot d\bar{L} = \int_S \left[\bar{J} + \frac{\partial D}{\partial t}\right] \cdot d\bar{S}} \quad \text{--- (iv)} \quad \text{(In integral form)}$$

* Apply strokes theorem to LHS of equ (4)

-> Convert line integral into surface integral.

$$\oint_L \bar{H} \cdot d\bar{L} = \int_S (\nabla \times \bar{H}) \cdot d\bar{S}$$

$$\int_S (\nabla\times\bar{H})\cdot d\bar{S} = \int_S \left[\bar{J} + \frac{\partial D}{\partial t}\right] d\bar{S}$$

$$\boxed{\nabla\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}} \quad \text{---} (v)$$

Point form or differential form.

[2] From faraday's Law :-

According to faraday's Law,

The induced EMF is given by,

$$\boxed{\oint_L \bar{E}\cdot d\bar{L} = -\int_S \frac{\partial\bar{B}}{\partial t}\cdot dS} \quad \text{---} (i)$$

integral form,

* Applying Stroke's theorem to LHS of equ (i),

$$\int_L \bar{E}\cdot d\bar{L} = \int_S (\nabla\times\bar{E})\cdot d\bar{S}$$

$$\int_S (\nabla\times\bar{E}) dS = -\int_S \frac{\partial\bar{B}}{\partial t}\cdot dS$$

$$\boxed{\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}}$$

Point form or differential form.

[3] From Gauss's Law,

(i) For electric field,

* According to gauss law

$$\boxed{\int_S \bar{D}\cdot dS = \int Q_{enc}} \quad \text{--} (1)$$

* Q_{enc} in terms of volume charge density ρ_v is given by.

$$\boxed{Q_{enc} = \int_V \rho_v dv} \quad \text{---} (ii)$$

$$\boxed{\int_S \bar{D}\cdot ds = \int_v \rho_v dv}$$ --- (3) integral form.

* Applying divergence theorem ~~to LHS of~~
  to LHS of equ (3).

→ Convert surface integral into volume integral

$$\oint_S \bar{D}\cdot d\bar{s} = \int_v \nabla\cdot\bar{D}\, dv$$

$$\int_v \nabla\cdot\bar{D}\, dv = \int_v \rho_v dv$$

$$\boxed{\nabla\cdot\bar{D} = \rho_v}$$ --- (iv)

Point form (or) differential form.

(ii) ~~Fr~~ For magnetic field,

* According to Gauss's law.

$$\boxed{\int_S \bar{B}\cdot ds = \phi}$$ --- (i)

For a closed surface,

$$\boxed{\oint_S \bar{B}\cdot dS = 0}$$ — (ii)

integral form; ~~[illegible]~~

* Applying divergence at equ (i),
* Convert Surface integral into Volume integral.

$$\int_S B \cdot d\bar{S} = \int_V \nabla \cdot \bar{B} \, dv$$

$$\int_V \nabla \cdot \bar{B} \, dv = 0$$

$$\boxed{\nabla \cdot \bar{B} = 0}$$ Point form (or) differential form.

* Maxwell's Equ for free Space :-

Free Space is a non- Conducting medium in which Volume charge density sow be zero and Conductibity σ (Sigma) = 0.

$\rho_v = 0$

$\sigma = 0$

Maxwell equ in free Space.

(A) Point form

(i) $\nabla \times E = \frac{-\partial \bar{B}}{t}$

(ii) $\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$

(iv $\nabla \cdot D = 0$

(v) $\nabla \cdot B = 0$

(B) Integral form.

(i) $\oint \bar{E} \cdot d\bar{L} = -\int \frac{\partial \bar{B}}{\partial t} \cdot ds$

(ii) $\oint \bar{H} d\bar{L} = \int_S \frac{\partial B}{\partial t} \cdot d\bar{S}$

(iii) $\oint \bar{D} \cdot d\bar{S} = 0$

(iv) $\oint \bar{B} \, d\bar{S} = 0$

Maxwell's equ for Good Conductor.

$J_C >>> J_D$

$\rho_v = 0$

for Good Conductor,

Maxwell equ in free Space.

(A) Point form

$$\nabla \times \bar{E} = \frac{-\partial B}{\partial t}$$

$$\nabla \times \bar{H} = \bar{J}$$

$$\nabla \cdot \bar{D} = 0$$

$$\nabla \cdot B = 0$$

(B) Integral form.

(i) $\oint \bar{E} \cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$

(ii) $\oint \bar{H} \cdot dL = I = \int_S \bar{J} \cdot dS$

(iii) $\oint \bar{D} \cdot dS = 0$

(iv) $\oint \bar{B}\, d\bar{S} = 0$

Maxwell equation for Harmonically Varaying fields:

=> The electric & magnetic field are varaying harmonaphically with time. The electric flux density is give by.

$$* \; \bar{D} = D_0 e^{j\omega t}$$

Taking Partial derivative

$$\frac{\partial \bar{D}}{\partial t} = D_0 e^{j\omega t} \cdot j\omega$$

$$\boxed{\frac{\partial \bar{D}}{\partial t} = j\omega \bar{D} = j\omega \varepsilon \bar{E}} \quad \bar{D} = \varepsilon \bar{E}$$

Similarly the magnetic field density can be,

$$* \; \bar{B} = B_0 e^{j\omega t}$$

Taking partial derivate,

$$\frac{\partial \bar{B}}{\partial t} = B_0 e^{j\omega t} \cdot j\omega$$

$$\boxed{\frac{\partial \bar{B}}{\partial t} = j\omega B = j\omega \mu \bar{H}} \quad (B = \mu \bar{H})$$

Maxwell equ in :-

(A) Point form

(i) $\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} = -j\omega B$

(ii) $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

(iii) $\nabla \cdot \bar{D} = \rho_v$

(iv) $\nabla \cdot B = 0$

(B) integral form

(i) $\oint \bar{E} \cdot d\bar{L} = -\int_S \left(\frac{\partial \bar{B}}{\partial t}\right) d\bar{s}$

(ii) $\oint \bar{H} \cdot d\bar{L} = I = \int_S \left(\bar{J}_c + \frac{\partial \bar{D}}{\partial t}\right) \cdot d\bar{s}$

(iii) $\oint \bar{D} \cdot ds = \oint \rho_v dv$

(iv) $\oint \bar{B} \cdot \bar{s} = 0$